Thermal Transient Analysis to Locate Hydrate Growth and Deposition in Natural Gas and Gas-Condensate Vertical Wells

By

Oluwatoyin Akinsete¹*, Kessiena Arhata* And Sunday Isehunwa*

ABSTRACT

Transient flow of gas and gas-condensate in wells is a problem of industrial interest. Studies abound in the use of pressure transient models in the analysis of hydrate formation, growth and deposition in these wells but no study in the use of thermal transient model in the evaluation of these crystalline solids in gas and gas-condensate in wells. Partial and/or complete well blockages stem out from the deposition of hydrates and other solids, but hydrate deposition happens to be the one having the most significant effect in the blockage of these wells leading to reducing flow capacity or total loss in productivity.

This work therefore aimed at developing and using thermal transient analytical model to locate hydrates growth and deposition in vertical wells. To accurately do this, the conservation of material, momentum and energy conservation which were the governing equations was applied to solve the transient flow of gas and gas-condensate in pipelines with respect to their diameter. The application of these conservation equations resulted into nonlinear hyperbolic equations, which were arranged in a canonical form and solved analytically using method of characteristics.

Developed model revealed that hydrate formation or growth and deposition in these fluids well is a function of fluid and solid mechanics, changes in fluid and surrounding conditions, fluid composition and flow rate variation. Solutions of the resulting equations include consideration of flow rate, shear stress and compressibility factor to hydrate growth along pipeline. The results were compared with the field and literature data and a sensitivity analysis was carried out to determine the effects of these varied parameters in the formation, growth and deposition of hydrates in gas and gas-condensate wells.

Key words: Thermal transient model, Hydrate, Gas, Gas-Condensate, Vertical wells

1. INTRODUCTION

Hydrates are ice-like crystalline solids in nature formed when gas (guest) molecules contact free water (host) at low temperature and high pressure.² They are also known as clathrates to distinguish them from stoichiometric hydrates found in inorganic chemistry.³ The cages are

¹ Corresponding Author: e-mail: kunle2223@yahoo.com
² Authors address: Department of Petroleum Engineering, Faculty of Technology, University of Ibadan, Nigeria
stabilized by Van der Waals forces between the water molecules and the enclathrated guest molecule. The host molecules in crystal hydrate mostly water although other liquids such as Freon-12 can also serve as host molecules. Light hydrocarbons such as methane, ethane, e.t.c or compounds such as carbon dioxide and hydrogen sulfide makes up the guest molecules.

Von Stackelberg et al., in 1951 studied the hydrate structure using x-ray diffraction methods, their work along with works by Claussen identified two hydrate structures, structure I and structure II. Ripmeester et al., in 1987 discovered a third type of hydrate structure, structure H using NMR technique (Fig. 1).

Fig. 1: Gas Hydrate Structure. Source: (https://en.m.wikipedia.org/wiki/clathrate_hydrate)

put forward a theoretical and conceptual model to describe hydrate formation stages as nucleation, growth, agglomeration, and plugging on the basis of crystallization theories. In 2004, Sinquin et al. showed that the condition favouring hydrate formation include high pressure (usually greater than 30 bar) and low temperature (usually less than 20°C) and is dependent on the fluid composition. Also the presence of hydrate formers such as C\textsubscript{n}H\textsubscript{2n+2}, n = 1-5, CO\textsubscript{2}, H\textsubscript{2}S, Freon 12 (dichlorodifluromethane), Ethylene, Acetylene, Propylene, Propyne, Cyclopropane, Cis-2-butene, iso-2-butene etc also enhance the formation of hydrates.

---

Before now, studies\textsuperscript{10,11,12,13,14} in the literature used pressure transient under isothermal condition in the analysis of hydrate formation tracking, location and deposition in pipelines. The primary thrust of this work was to develop a simple thermal transient analytical model to predict hydrate growth in natural gas pipeline systems. The developed model was used to: thoroughly check the behaviour of hydrate growth for different gas compositions, investigate the effects of flow rate, shear stress, compressibility factor (or Z-factor), and examine the influence of increasing temperature on the formation of gas hydrates. Akinsete and Isehunwa\textsuperscript{15} in 2015 developed pressure transient analytical model using conservations of mass and momentum as the governing equations, their work showed the effects of different methane fraction component behaviour in natural gas stream to hydrate growth and blockage severity. They discovered that a simple analytical method can be used to resolve the nonlinear partial differential equation that describes the fluid flow problems in natural gas pipeline systems. This present work is an improvement on the study of Akinsete and Isehunwa\textsuperscript{16} by including the conservation of energy to make the present work more robust and help examine the effect of temperature on the hydrate formation and deposition in gas and gas-condensate vertical wells.

2. \textbf{Mathematical Model Development}

The dynamics incorporated in the formation of these solid particles are governed by the single phase hydrodynamics equation developed from mass, momentum and energy conservations.

Mass Conservation Equation

\[
\frac{\partial \ell}{\partial t} + \frac{\partial \ell u}{\partial x} = 0
\]

Momentum Conservation Equation

\[
\frac{\partial (\ell u)}{\partial x} + \frac{\partial (\ell u^2 + p)}{\partial x} = \gamma
\]


\textsuperscript{14} Akinsete, O.O., and S.O. Isehunwa, 2015, Novel analytical model for predicting hydrate formation onset pressures in natural gas pipeline systems: \textit{Journal of Characterization and Development of Novel Materials}.

\textsuperscript{15} Akinsete, O.O., and S.O. Isehunwa, \textit{Op. Cit.}

Where \( \Psi = -\tau_D - \tau_K - \ell g \sin \theta \)

Energy Conservation Equation

\[
\frac{\partial \Psi}{\partial t} + \frac{\partial \Psi u}{\partial x} = \beta
\]

Where, \( \beta = \frac{\ell g - \ell u g \sin \theta}{c} \)

**Characteristic Method of Solution**

This work made use of the characteristic based solution method developed by the authors,\(^{17,18,19,20}\) it was used in solving single phase flow problems. The system of governing equations written in canonical form is as follows:

\[
\frac{\partial \Psi}{\partial t} + \frac{\partial F}{\partial x} + D = 0
\]

In equation 4, \( \Psi \) is the vector of the composite variables, \( F \) contains all terms with a derivative with respect to the x-direction and \( D \) contains all other terms i.e.

\[
\Psi = \begin{bmatrix} \alpha \ell \\ \alpha \ell u \\ \alpha \ell T \end{bmatrix}, \quad F = \begin{bmatrix} \alpha \ell u \\ \alpha \ell u^2 + \alpha p \\ \alpha \ell u T \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ \Psi \\ \beta \end{bmatrix}
\]

In primitive form equation 4 becomes:

\[
\frac{\partial \Psi}{\partial t} + \hat{A} \frac{\partial \Psi}{\partial x} + \hat{C} = 0
\]

Where \( \Psi = \begin{bmatrix} P \\ T \\ u \end{bmatrix} \)


Multiply equation 6 by transformation matrix J.

\[ J \frac{\partial \mathbf{u}}{\partial t} + JA \frac{\partial \mathbf{u}}{\partial x} + JC = 0 \]  

Where J, a Jacobian matrix is given as:

\[
J = \begin{bmatrix}
\alpha \frac{\partial \ell}{\partial p} & \alpha \frac{\partial \ell}{\partial t} & 0 \\
\alpha u \frac{\partial \ell}{\partial p} & \alpha u \frac{\partial \ell}{\partial t} & \alpha \ell \\
\alpha T \frac{\partial \ell}{\partial p} - \alpha \ell \frac{\partial \ell}{\partial t} & \alpha \ell + \alpha T \frac{\partial \ell}{\partial t} & 0
\end{bmatrix}
\]

And the inverse of J is given as:

\[
J^{-1} = \begin{bmatrix}
\left( \ell + T \frac{\partial \ell}{\partial t} \right) / 2 \alpha \ell \frac{\partial \ell}{\partial p} & 0 & -\frac{\partial \ell}{\partial t} / 2 \alpha \ell \frac{\partial \ell}{\partial p} \\
\left( \ell \frac{\partial \ell}{\partial p} - T \frac{\partial \ell}{\partial t} \right) / 2 \alpha \ell \frac{\partial \ell}{\partial p} & 0 & 1 / 2 \alpha \ell \\
-\frac{1}{\alpha \ell} & 1 / (\alpha \ell) & 0
\end{bmatrix}
\]

Introducing time-dependent primitive boundary condition \( \mathbf{h} \) by Bjorhus\(^2\), we have:

\[ \frac{\partial \mathbf{u}}{\partial t} + \nabla \mathbf{h} + \mathbf{\zeta} = 0 \]

Where \( V \) is the eigenvector equation which is given as:

\( (\mathbf{A} - \lambda I)X = 0 \)

Applying the mathematics of eigenvalue and eigenvector where \( X \) is a non-trivial solution the equation 11 is given as:

\[ |\mathbf{A} - \lambda I| = 0 \]

Where $I$ is the identity matrix which is taken to be the same size as $\bar{A}$. The matrix $\bar{A}$ is given as:

$$\bar{A} = \begin{bmatrix}
\frac{(u_T(1 + \frac{\partial T}{\partial t}))}{2\ell} & 0 & \frac{1}{2\ell} \\
\frac{\partial P}{2\ell} & u & \frac{\partial P}{2\ell} \\
\frac{\partial P}{\ell} & \frac{\partial P}{\ell} & u
\end{bmatrix}$$

The characteristic velocity which is also the eigenvalue is given as:

$$\lambda_1 = c - u, \quad \lambda_2 = u, \quad \lambda_3 = c + u$$

The eigenvector matrix is given as:

$$V = \begin{bmatrix}
\frac{-\ell c}{2} & -\frac{\partial P}{\partial T} & \frac{-\ell c}{2} \\
\frac{-\ell c \partial T}{\partial P} & \frac{1}{2} & \frac{-\ell c \partial T}{\partial P} \\
\frac{1}{2} & 0 & \frac{1}{2}
\end{bmatrix}$$

In matrix form where $c = 0$, the equation 10 is given as:

$$\frac{\partial}{\partial t}\begin{bmatrix} P \\ T \end{bmatrix} + \begin{bmatrix}
\frac{(\ell^2 \frac{\partial T}{\partial P})}{2\ell} & -\frac{\partial P}{\partial T} & \frac{(\ell^2 \frac{\partial T}{\partial P})}{2\ell} \\
\frac{\partial P}{2\ell} & \frac{1}{2} & \frac{\partial P}{2\ell} \\
\frac{1}{2} & 0 & \frac{1}{2}
\end{bmatrix} \begin{bmatrix} \bar{h}_1 \\ \bar{h}_2 \\ \bar{h}_3 \end{bmatrix} = 0$$

Expanding the equation 16 and solving for $\bar{h}_1$, $\bar{h}_2$, and $\bar{h}_3$ we have:

$$\frac{\partial P}{\partial t} - \frac{(\ell^2 \frac{\partial T}{\partial P})}{2\ell} \bar{h}_1 - \frac{\partial P}{\partial T} \bar{h}_2 + \frac{(\ell^2 \frac{\partial T}{\partial P})}{2\ell} \bar{h}_3 = 0$$
Substituting the values of $\bar{h}_1$, $\bar{h}_2$, and $\bar{h}_3$ into equation 17 gives:

$$2 \frac{\partial P}{\partial t} - \left( \frac{\partial}{\partial t} \sqrt{\frac{\partial}{\partial P}} \right) + 2 \left( \frac{\partial}{\partial P} \right) \left( \left( -\frac{\partial}{\partial t} + 4 \frac{\partial}{\partial P} \right) \frac{\partial P}{\partial x} + \left( \frac{\partial}{\partial t} \sqrt{\frac{\partial}{\partial P}} \right) \frac{\partial x}{\partial x} \right) = 0$$

Introducing the acoustic speed of sound equation (22) to equation (21)

$$c^2 = \frac{P}{\ell} = \frac{ZRT}{M}$$

Gives:
Comparing the constant terms in equations 4 and 7, gives:

\[ \frac{\partial u}{\partial x} = \Psi^{-1} D \]

This is represented in matrix form as:

\[
\begin{bmatrix}
\frac{\partial P}{\partial x} \\
\frac{\partial T}{\partial x} \\
\frac{\partial u}{\partial x}
\end{bmatrix} = \frac{1}{\varepsilon} \begin{bmatrix}
\alpha^2 \ell^2 u \Psi + \alpha^2 \ell \beta \left( u^2 \frac{\partial \ell}{\partial T} - \frac{\partial P}{\partial T} \right) \\
\alpha^2 \ell^2 u \Psi \frac{\partial T}{\partial P} + \alpha^2 \ell \beta \left( 1 - \frac{u^2}{c^2} \right) \\
-2\alpha^2 \ell u^2 \frac{\partial u}{\partial x}
\end{bmatrix}
\]

Expanding results in:

\[ \varepsilon \frac{\partial T}{\partial x} = \alpha^2 \ell^2 u \frac{\partial T}{\partial P} (\tau + \alpha \ell g \sin \theta) + \alpha^2 \ell \left( \frac{\alpha g - \alpha \mu g \sin \theta}{k} \right) \left( 1 - \frac{u^2}{c^2} \right) \]

Where \( \varepsilon \) is \( |\Psi| \) and is given as:

\[ \varepsilon = 2\alpha^2 \ell^2 u \left( 1 - \frac{u^2}{c^2} \right) \]

Let the specific heat be represented by \( k \), hence;

\[ \Psi = \tau + \alpha \ell g \sin \theta, \quad \beta = \frac{\alpha g - \alpha \mu g \sin \theta}{k} \] and \( \tau = \frac{f \alpha \ell u^2}{2D} \)

Substituting equation 27 into equation 26 and rearranging, gives hydrate volume fraction equation 28

\[ \alpha = \frac{\ell^2 u \tau (T_{out} - T_{in})/(P_{out} - P_{in}) L}{\left(2\ell^2 u(1 - \frac{u^2}{c^2}) - \ell L \left( \frac{\ell g - \ell \mu g \sin \theta}{k} \right) \left(1 - \frac{u^2}{c^2} \right) - \ell^8 \ell g L \sin \theta (T_{out} - T_{in})/(P_{out} - P_{in}) \right)} \]
The hydrate volume fraction equation 28 is a function of fluid and solid mechanics, changes in fluid and surrounding conditions, fluid composition and flow rate variation. The equation was used to determine the effect of temperature, shear stress, flow rate, and gas deviation factor on hydrate formation and deposition in pipelines. Field and literature physical and compositional data were used in the analysis.

3. RESULTS AND DISCUSSION

Dry gas and gas-condensate samples of specific gravity of 0.60±0.01 and 0.62±0.01 respectively and methane fractions of 0.90±0.01 and 0.80±0.05 for dry gas and gas-condensate respectively were used.

Effect of Temperature

Table 1 shows how hydrate formation for both field and literature data decreases with increasing temperature.

Table 1: Temperature effect on hydrate growth.

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Dry Gas A</th>
<th>Dry Gas B</th>
<th>Gas-Condensate A</th>
<th>Gas-Condensate B</th>
<th>Gas-Condensate C</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10.5</td>
<td>1.784</td>
<td>1.790</td>
<td>1.821</td>
<td>2.123</td>
<td>3.446</td>
</tr>
<tr>
<td>-5.8</td>
<td>1.419</td>
<td>1.423</td>
<td>1.444</td>
<td>1.631</td>
<td>2.331</td>
</tr>
<tr>
<td>-0.4</td>
<td>1.108</td>
<td>1.111</td>
<td>1.123</td>
<td>1.236</td>
<td>1.612</td>
</tr>
<tr>
<td>8.3</td>
<td>0.752</td>
<td>0.753</td>
<td>0.759</td>
<td>0.812</td>
<td>0.968</td>
</tr>
<tr>
<td>25.6</td>
<td>0.324</td>
<td>0.324</td>
<td>0.326</td>
<td>0.337</td>
<td>0.367</td>
</tr>
<tr>
<td>35.8</td>
<td>0.163</td>
<td>0.163</td>
<td>0.164</td>
<td>0.167</td>
<td>0.177</td>
</tr>
</tbody>
</table>

At pipeline length of 1000 miles, an increase in temperature from -10.5, -5.8, -0.4, 8.3, 25.6, to 35.8 °C, led to a corresponding decrease in hydrate growth of 1.783511, 1.419403, 1.107556, 0.751572, 0.323929, and 0.163174 Å for dry gas and that of others are shown in the table. Figure 2 goes further to show that increasing temperature leads to decreasing hydrate formation.
Flow rate was varied in the model while keeping other parameters constant to determine the effect of it on hydrate growth. It was seen that at a low temperature of -10.5 °C, flow rate was increased from 1.981e8 scf to 2.214e8 scf and these led to a decrease in hydrate growth from 1.76223 to 1.148993 Å at pipeline length of 1000 mm. The result are seen in Table 2 and Figure 3 gives a picture of the decrease in the hydrate growth at 1000 miles.

**Table 2: Effect of flow rate**

<table>
<thead>
<tr>
<th>Temperature °C</th>
<th>Flow Rate (MMscf)</th>
<th>Flow Rate (MMscf)</th>
<th>Flow Rate (MMscf)</th>
<th>Flow Rate (MMscf)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>198.1</td>
<td>221.4</td>
<td>252.3</td>
<td>274.7</td>
</tr>
<tr>
<td>-10.5</td>
<td>1.800</td>
<td>1.162</td>
<td>0.871</td>
<td>0.769</td>
</tr>
<tr>
<td>-5.8</td>
<td>1.426</td>
<td>0.987</td>
<td>0.766</td>
<td>0.684</td>
</tr>
<tr>
<td>8.3</td>
<td>0.750</td>
<td>0.597</td>
<td>0.501</td>
<td>0.461</td>
</tr>
<tr>
<td>35.8</td>
<td>0.162</td>
<td>0.148</td>
<td>0.137</td>
<td>0.132</td>
</tr>
</tbody>
</table>
Effect of Shear Stress.

Table 3 shows the growth of gas hydrate along a 1000mm pipeline length at varying shear stress for Niger delta Dry-Gas A compositional data at low temperature of -10.5 °C. It can be seen that as the shear stress was increased from 2.982psi to 5.532psi the hydrate growth also increased from 0.695515 to 0.755924 Å. This is clearly shown in Figure 4, where hydrate growth increases with increase in shear stress.

Table 3: Effect of shear stress

<table>
<thead>
<tr>
<th>Temperature °C</th>
<th>Shear Stress (psi)</th>
<th>2.982</th>
<th>3.241</th>
<th>4.426</th>
<th>5.523</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10.5</td>
<td>0.696</td>
<td>0.756</td>
<td>1.032</td>
<td>1.288</td>
<td></td>
</tr>
<tr>
<td>-5.8</td>
<td>0.554</td>
<td>0.602</td>
<td>0.822</td>
<td>1.025</td>
<td></td>
</tr>
<tr>
<td>8.3</td>
<td>0.293</td>
<td>0.319</td>
<td>0.435</td>
<td>0.543</td>
<td></td>
</tr>
<tr>
<td>35.8</td>
<td>0.064</td>
<td>0.069</td>
<td>0.094</td>
<td>0.118</td>
<td></td>
</tr>
</tbody>
</table>
Effect of Gas Deviation Factor (or Z-factor)

Gas deviation factor also known as compressibility factor (Z-factor) which is a function of both absolute temperature and absolute pressure for gas composition was varied to determine the effect on the solid hydrate formation. Table 4 shows the growth of gas hydrate along a 1000mm pipeline length at varying Z-factor. Figure 5 shows that as gas deviation factor is varied from 0.684 to 0.767, the solid hydrate fraction is decreased from 2.054927 to 1.248289 Å at temperature of -10.5 °C.

Table 4: Effect of Z-Factor

<table>
<thead>
<tr>
<th>Temperature °C</th>
<th>0.684</th>
<th>0.767</th>
<th>0.823</th>
<th>0.942</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10.5</td>
<td>2.587</td>
<td>1.762</td>
<td>1.495</td>
<td>1.184</td>
</tr>
<tr>
<td>-5.8</td>
<td>0.729</td>
<td>0.681</td>
<td>0.656</td>
<td>0.618</td>
</tr>
<tr>
<td>8.3</td>
<td>0.483</td>
<td>0.460</td>
<td>0.448</td>
<td>0.428</td>
</tr>
<tr>
<td>35.8</td>
<td>0.135</td>
<td>0.132</td>
<td>0.130</td>
<td>0.128</td>
</tr>
</tbody>
</table>
4. CONCLUSION

A thermal transient model was developed to predict the formation of hydrate precipitation; growth and deposition in gas pipeline. The developed model was found to be a function of fluid and solid mechanics, changes in fluid and surrounding conditions, fluid composition and flow rate variation along dry gas and gas-condensate pipeline systems. Of these factors, changes in fluid and surrounding conditions (i.e. temperature) have the most effect and others have a pronounced effect on hydrate precipitation, growth and deposition along pipelines.

Figure 5: Hydrate thickness vs. Z-factor