Optimization of Kartoatmodjo and Schmidt Oil Formation Volume Factor Correlation with Particle Swarm Optimization Algorithm

By

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Abstract
Knowledge of reservoir fluid properties is very important in reservoir engineering calculations such as reserves estimates, material balance equation, and numerical reservoir stimulation etc. Laboratory measurements are the most dependable methods for pressure-volume-temperature (PVT) analysis of reservoir fluid properties. During reservoir performance evaluation, the measurements of PVT data are not readily available, hence engineers resort to the use of existing empirical correlations to obtain PVT data. Often time, the data obtained from correlations do not agree with the experimental data, thereby yielding a considerable amount of error. To correct that error in this study, Kartoatmodjo and Schmidt correlation for estimating the oil formation volume factor was optimized using the particle swarm optimization (PSO) algorithm to minimize the error in estimating the oil formation volume factor from Kartoatmodjo and Schmidt correlation at various depletion pressure. The optimized correlation is a function of bubble point pressure, API gravity, gas gravity and reservoir temperature. Therefore, to validate the optimized correlation, PVT data from differential liberation test in the laboratory was used as case a study and the result obtained with the optimized correlation matches closely with the experimentally values at various depletion pressure. Also, the Kartoatmodjo and Schmidt optimized correlation was validated with Standing, Kartoatmodjo and Schmidt, Glaso and Petrosky-Fashad correlations. The optimized Kartoatmodjo and Schmidt gave the least average relative error of ±0.00065 and the highest correlation coefficient of 0.99996 after 154th successive iterations with the PSO algorithm.

Keywords: differential liberation test, gas-oil ratio, oil formation volume factor, PSO, PVT correlations, reservoir fluid sampling, PVT analysis

INTRODUCTION
Pressure-volume-temperature (PVT) data are key input parameters for most reservoir engineering calculations such as material balance, hydrocarbon reserves estimation, reservoir performance prediction, enhanced oil recovery scheme, design and optimization of production systems (Ahmed1) etc. The material balance equation is a strong function of PVT data. According to Ikiensikimama2, the estimation of reserves and the design of the best depletion strategy are only feasible when realistic and reasonably accurate values of the reservoir fluid properties are available. At the earlier stages of a well, it can be difficult or economically impractical to obtain reliable measurements of PVT data. Therefore, in a scenario where the reservoir fluid samples are available, they can be subjected to PVT analysis to determine the key fluid properties. According to Ikiensikimama and Egbe3, PVT analysis are usually performed at reservoir temperature.

PVT data can either be obtained from laboratory analysis or generated from existing correlations developed from different regions. In accordance with the statement made by Bon

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et al.\textsuperscript{4}, that for oil reservoirs, the PVT analysis is conducted on samples taken when the reservoir pressure is above the bubble point. Hence, prior to carrying out the laboratory analysis, the reservoir fluid is first sampled either at the subsurface or surface (wellhead/separator) and thereafter conveyed to the laboratory to check for the quality, consistency and possible leakage. Table 1 shows the types of PVT experiments conducted in the laboratory and the respective data obtained.

Table 1: Types of PVT experiment and data derived

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Data Obtain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant composition expansion (black oil and compositional)</td>
<td>Relative volume, Vapor Z factor, Liquid Drop Out</td>
</tr>
<tr>
<td>Differential Liberation/Vaporization (black oil only)</td>
<td>Vapour Z factor, Liquid density, Gas-Oil ratio, Relative volume (Formation volume factor), Gas gravity, Liquid viscosity, Vapor viscosity</td>
</tr>
<tr>
<td>Constant Volume Depletion (compositional only)</td>
<td>Retrograde liquid drops out, Cumulative fluid produced, Vapor Z factor, Specific gravity of produced fluid plus, Mole weight of produced fluid plus, Final weight of produced fluid plus &amp; Produced vapor composition.</td>
</tr>
<tr>
<td>Separator test (black oil and compositional)</td>
<td>Gas-Oil ratio &amp; Stock tank formation volume factor</td>
</tr>
<tr>
<td>And others such as Saturation pressure (Dew point) at reservoir temperature, viscosity measurement &amp; compositional analysis.</td>
<td></td>
</tr>
</tbody>
</table>

Furthermore, carrying the experiments listed in Table 1 are usually expensive and time-consuming, thus, Engineers in the field resorted to using existing correlation to estimate these properties. Several correlations have been developed in literature to estimate PVT properties such as gas-oil ratio and oil formation volume factor. This study focuses on oil formation volume factor only and some of the existing correlations used in the oil and gas industry are:

Standing\textsuperscript{5} correlation for oil formation volume factor (oil FVF) expressed as:

$$B_o = 0.972 + 0.000147 \left[ R_s \left( \frac{Y_g}{Y_o} \right)^{0.5} + 1.25T \right]^{1.175}$$  \hspace{1cm} (1)

Glaso\textsuperscript{6} proposed a correlation for estimating oil formation volume factor as a function of the API gravity, solution gas-oil ratio \(R\), temperature, and gas specific gravity given by:

$$B_o = 1.0 + 10^A$$  \hspace{1cm} (2)

Where


Optimization of Kartoatmodjo and Schmidt

\[ A = -6.58511 + 2.91329 \log \left( R_s \frac{Y_g}{Y_o}^{0.526} + 0.9867 \right) \]

\[ -0.27683 \left[ \log \left( R_s \frac{Y_g}{Y_o}^{0.526} + 0.9867 \right) \right]^2 \] (3)

Al-Marhoun\(^7\) developed a correlation for estimating oil formation volume factor for Middle East crude oil. These correlations were based on a database of 69 bottom hole fluid samples and expressed as a function of reservoir temperature, solution GOR, gas gravity and oil gravity.

\[ B_o = 0.497069 + 0.000826863(T + 459.67) + 0.00182594F + 0.318099 \times 10^{-6}F^2 \] (4)

Where

\[ F = R_s^{0.74239}Y_g^{0.323294}Y_o^{-1.202040} \] (4a)

Petrosky and Farshad\(^8\) used a nonlinear multiple regression software to develop an oil formation volume factor. The authors constructed a PVT database from 81 laboratory analyses from the Gulf of Mexico crude oil system. Petrosky and Farshad proposed the following expression

\[ B_o = 1.0113 + 7.204 \times 10^{-5} \left[ R_s^{0.3738} \left( \frac{Y_g^{0.2914}}{Y_o^{0.6265}} \right) + 0.2467T^{0.5371} \right]^{3.0936} \] (5)

Kartoatmodjo and Schmidt\(^9\) developed a new set of empirical correlation based on a large collection of data developed from all over the world. Their equation is given by

\[ B_o = 0.98496 + 0.0001 \times \left[ R_s^{0.755}Y_g^{0.25}Y_o^{-1.5} + 0.457 \right]^{1.50} \] (6)

It is imperative to note that most of these correlations yield accurate results when applied at the bubble point pressure. However, for pressures below the bubble point pressure, the computed PVT properties may yield a considerable error, which make the application of these correlations in reservoir studies or material balance equation not accurate because the values obtained from correlations are not equal to the experimental data. Hence in order to minimize error in estimating oil formation volume factor from correlations, an optimization algorithm is required. Few of these algorithms are gradient descend, evolutionary algorithms, particle swamp optimization tool etc. Therefore, the aim of this study is to minimize the error between the experimented results of oil formation volume factor (Bo) and the result from existing correlations. To achieve the aim of this study, Kartoatmodjo and Schmidt oil formation volume factor correlation was modified with PSO.

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METHODOLOGY
The methodological approach of this study is presented in Figure 1.

Figure 1: Methodology of study

INPUT DATA
Table 2: Differential liberation test data

<table>
<thead>
<tr>
<th>Pressure (psig)</th>
<th>Formation volume factor (rb/stb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2405</td>
<td>1.568</td>
</tr>
<tr>
<td>2200</td>
<td>1.579</td>
</tr>
<tr>
<td>1950</td>
<td>1.476</td>
</tr>
<tr>
<td>1700</td>
<td>1.433</td>
</tr>
<tr>
<td>1450</td>
<td>1.392</td>
</tr>
<tr>
<td>1200</td>
<td>1.353</td>
</tr>
<tr>
<td>950</td>
<td>1.313</td>
</tr>
<tr>
<td>700</td>
<td>1.269</td>
</tr>
<tr>
<td>450</td>
<td>1.219</td>
</tr>
<tr>
<td>200</td>
<td>1.182</td>
</tr>
</tbody>
</table>
Optimization of Kartoatmodjo and Schmidt

**Generation of Objective function**
The objective functions developed in this study is based on Kartoatmodjo and Schmidt correlation for oil formation volume factor below the bubble point pressure.

\[
B_o = 0.98496 + 0.0001 \times \left( R_s^{0.755} Y_g^{0.25} Y_o^{-1.5} + 0.45T \right)^{1.50}
\]

(7)

Hence

\[
B_o \text{estimated} = A + B \times \left( R_s^{0.755} Y_g^{0.25} Y_o^{-1.5} + 0.45T \right)^C
\]

(8)

Hence objective function can be defined as

\[
f(x) = \sum |B_o \text{exptal} - A + B \times \left( R_s^{0.755} Y_g^{0.25} Y_o^{-1.5} + 0.45T \right)^C| = \text{min}
\]

Where

\(B_o \text{exptal}\) is the values obtained from laboratory PVT analysis on a nearby field;

A, B, and C is a minimization constant to be determined by PSO optimization tool;

\(B_o \text{estimated}\) is the estimated values obtained from PSO optimization tool;

**PSO ALGORITHM STEPS**

Step 1: Choose the number of particles
Step 2: Initialize the initial positions of the particles
Step 3: Evaluate the objective function at the initial positions
Step 4: Set the iteration number as \(t = i+1\)
Step 5: Find the personal best for each particle
Step 6: Find the global best
Step 7: find the velocities of the particles
Step 8: Find the new values of the particles position
Step 9: Find the objective function values of step 6
Step 10: Stopping criterion:

If the terminal rule is satisfied, go to step 4, otherwise stop the iteration and output the results

Therefore, the iterative process of the PSO algorithm is represented in Figure 2.


**Figure 2:** Workflow algorithm for the PSO (Gbest PSO)

1. **Start**
2. Initialize position $x_{ij}^0$, $c_1, c_2$, velocity $V_{ij}^0$, evaluate $f_{ij}^0$
3. $D=$max number of dimension, $P=$max number of particles, $N=$max number of iterations
4. $t=0$
5. Choose $r_1^i, r_2^j$
6. $l=1$
7. $j=1$
8. $V_{ij}^{t+1} = V_{ij}^t + C_1 r_1^i [p_{best,i}^t - x_{ij}^t] + C_2 r_2^j [G_{best} - x_{ij}^t]$
9. $x_{ij}^{t+1} = x_{ij}^t + V_{ij}^{t+1}$
10. $l = l + 1$
11. $j = j + 1$
12. $t = t + 1$
13. $J < D$ (Yes/NO)
14. $i < P$ (Yes/NO)
15. Evaluate $f_{ij}^t$ using $x_{ij}^t$
16. $f_{ij}^t \leq f_{best,i}$ (Yes/NO)
17. $f_{ij}^t \leq f_{gbest}$ (Yes/NO)
18. $t \leq N$ (Yes/NO)

**Stop**

**Notes:**
- $p_{best,i}$: Personal best position of particle $i$
- $G_{best}$: Global best position of the swarm
- $C_1, C_2$: Cognitive and social coefficients
- $r_1^i, r_2^j$: Random numbers
- $x_{ij}^t$: Position of particle $i$ at iteration $t$
- $V_{ij}^t$: Velocity of particle $i$ at iteration $t$
- $f_{ij}^t$: Fitness value of particle $i$ at iteration $t$
- $f_{best,i}$: Best fitness value of particle $i$
- $f_{gbest}$: Best fitness value of the swarm
RESULTS AND DISCUSSION
The PSO algorithm as shown in Figure 2, is an iterative process that cannot be done manually, thus, it was programmed in Microsoft Excel to get the global best value for the constant A, B, and C respectively as shown in Figures A1 – A3 in appendix A.

Convergence criteria is a phenomenon of PSO in which all particles tend to converge to a single value as shown by Figures A1 – A3 respectively where all three particles maintain a single value at the 154th iterations. The modified Kartoatmodjo and Schmidt’s oil FVF correlation parameters are presented in Table 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Oil FVF @ 154th iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.003330294</td>
</tr>
<tr>
<td>B</td>
<td>0.000157176</td>
</tr>
<tr>
<td>C</td>
<td>1.41992552</td>
</tr>
</tbody>
</table>

Therefore, the modified Kartoatmodjo and Schmidt’s oil FVF correlation is given in Equations 10.

\[ B_{\text{optimized}} = 1.003330294 + 0.000157176 \times \left( R_2^{0.755} y_g^{0.25} y_o^{1.5} + 0.45 T \right)^{1.41992552} \]  

The results obtained from the existing oil formation volume factor correlations and the modified Kartoatmodjo and Schmidt’s oil FVF correlation with the PSO algorithm for this study are presented in Tables 4. The result clearly shows that the modified correlation performed better the other four correlations for the PVT laboratory data given in this study.

Table 4: Result of gas solubility (scf/stb) from existing correlations

<table>
<thead>
<tr>
<th>Pressure (bbl/stb)</th>
<th>Experimental</th>
<th>Kartoatmodjo and Schmidt</th>
<th>Petrosky-Fashad</th>
<th>Glaso</th>
<th>Standing</th>
<th>This Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>2405</td>
<td>1.568</td>
<td>1.5967</td>
<td>1.5967</td>
<td>1.5805</td>
<td>1.1835</td>
<td>1.5997</td>
</tr>
<tr>
<td>2200</td>
<td>1.519</td>
<td>1.5404</td>
<td>1.5404</td>
<td>1.5171</td>
<td>1.1675</td>
<td>1.5190</td>
</tr>
<tr>
<td>1950</td>
<td>1.476</td>
<td>1.4898</td>
<td>1.4898</td>
<td>1.4604</td>
<td>1.1531</td>
<td>1.4762</td>
</tr>
<tr>
<td>1700</td>
<td>1.433</td>
<td>1.4395</td>
<td>1.4395</td>
<td>1.4047</td>
<td>1.1388</td>
<td>1.4338</td>
</tr>
<tr>
<td>1450</td>
<td>1.392</td>
<td>1.3906</td>
<td>1.3906</td>
<td>1.3512</td>
<td>1.1250</td>
<td>1.3925</td>
</tr>
<tr>
<td>1200</td>
<td>1.353</td>
<td>1.3425</td>
<td>1.3425</td>
<td>1.2996</td>
<td>1.1115</td>
<td>1.3520</td>
</tr>
<tr>
<td>950</td>
<td>1.313</td>
<td>1.2951</td>
<td>1.2951</td>
<td>1.2502</td>
<td>1.0983</td>
<td>1.3120</td>
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<tr>
<td>700</td>
<td>1.269</td>
<td>1.2431</td>
<td>1.2431</td>
<td>1.1982</td>
<td>1.0841</td>
<td>1.2684</td>
</tr>
<tr>
<td>450</td>
<td>1.219</td>
<td>1.1842</td>
<td>1.1842</td>
<td>1.1434</td>
<td>1.0683</td>
<td>1.2190</td>
</tr>
<tr>
<td>200</td>
<td>1.182</td>
<td>1.1425</td>
<td>1.1425</td>
<td>1.1081</td>
<td>1.0575</td>
<td>1.1842</td>
</tr>
</tbody>
</table>

RESULT OF CROSS PLOT
To graphically illustrate the success of the existing correlations, a performance or cross plot was performed for the oil FVF correlations as shown in Figures A4 – A7 in Appendix A, which is a plot of the estimated values versus the experimental values. A 45° straight line was drawn on the cross plot on which the estimated value is equal to the experimental value. Fitting the straight line at 45° clearly indicates that, the closer the plotted data points are to this line, the
better the correlation. The result of Petrosky-Fashad, Glaso, Katoantmodjo-Schmidt and Standing correlations show a clear disparity from the experimented values, while the modified Katoantmodjo-Schmidt’s correlation gave a better match of the laboratory result at the different depletion pressures.

STATISTICAL ERROR ANALYSIS
From Table 5, the errors calculated by this study’s correlation are lower than the other three correlations. The modified correlation is most accurate for pressure below the bubble point pressure, yielding the least average percent relative error of -0.000650% and the highest correlation coefficient of 0.99996, which means that the sum of squares about the regression is minimal i.e. the unexplained variation by regression and thus agree with the theoretical concept that the best correlation should have the least average percent relative error and highest correlation coefficient between zero and one.

Table 5: Statistical Analysis of Correlations

<table>
<thead>
<tr>
<th>Statistical parameter</th>
<th>This study</th>
<th>Glaso correlation</th>
<th>Petrosky-Farshad correlation</th>
<th>Katoantmodjo-Schmidt correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average percent relative error</td>
<td>-0.000650</td>
<td>3.2</td>
<td>0.585</td>
<td>1.68</td>
</tr>
<tr>
<td>Sum squared residual</td>
<td>0.1480</td>
<td>0.2464</td>
<td>0.2090</td>
<td>0.165</td>
</tr>
<tr>
<td>Correlation coefficient</td>
<td>0.99996</td>
<td>0.9573</td>
<td>0.9873</td>
<td>0.9844</td>
</tr>
</tbody>
</table>

CONCLUSION
Based on the results of this evaluations, the following remarks are worth mentioning.

1. An empirical correlation for estimating the oil FVF at the bubble point pressure and below for black oils has been developed.
2. The result obtained with the data from the differential liberation test, shows that the correlation developed in this study performs better than Standing’s, Glaso’s, Petrosky’s and Katoantmodjo-Schmidt’s correlations. Hence, engineers can rely on the newly developed correlation to an extent after they have validated it with their field data because it is stated in literature that correlation performs better in the region it was developed.
3. The statistical result indicates a lower value of average relative error and a better coefficient of correlation for this study than for Standing’s, Glaso’s and Petrosky’s correlations at pressures below bubble point.
4. The particle swarm optimization tool achieves a better accuracy by minimizing the objective function generated in this study.

RECOMMENDATION

1. It is recommended that the developed black oil correlations should be optimized using the constant parameters in order to achieve a higher level of accuracy with field data from different geological region before application to a green field.
2. Further study should be done for oil formation volume factor above bubble point pressure.
3. The new correlation developed in this study should be further validated with several field data.
4. It is recommended that existing correlations for formation volume factor be coupled with differential liberation test experiment.
5. It is recommended that further studies considering black oils with significant amount of impurities such as carbon dioxide, hydrogen sulfide and nitrogen should be carried out.
6. Further work is recommended in developing a modification of Particle swarm optimization with faster convergence rate.
Appendix A

**Figure A1:** Constant A values from PSO iteration

**Figure A2:** Constant B values from PSO iteration
Figure A3: Constant C values from PSO iteration

Figure A4: Cross plots for Glaso oil formation volume factor correlation
Optimization of Kartoatmodjo and Schmidt

**Figure A5:** Cross plots of Petrosky and Farshad oil formation volume factor correlation

**Figure A6:** Cross plots of Kartoatmodjo and Schmidt Correlation oil formation volume factor correlation
Figure A7: Cross plots for formation volume factor for this study

**Error Analysis**
The following statistical means are used to determine the accuracy of the correlations:

Average percent relative error

$$E_r = \frac{1}{n} \sum_{1}^{n} E_i$$

$$E_i = \left( \frac{x_{exp} - x_{est}}{x_{exp}} \right)_i \times 100$$

Sum squared residual

$$SSR = \sum_{1}^{n} (x_{exp} - x_{est})^2$$

Correlation coefficient

$$r = \sqrt{1 - \left( \frac{\sum_{1}^{n} (x_{exp} - x_{est})^2}{\sum_{1}^{n} (x_{exp} - \bar{x})^2} \right)}$$

$$\bar{x} = \frac{1}{n} \sum_{1}^{n} (x_{exp})_i$$