Petroleum Technology Development Journal

Optimal Well Utilization under Uncertainty for Naturally Depleted and Water-flooded Reservoirs

By

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Published in:
Petroleum Technology Development Journal (ISSN 1595-9104)
An International Journal

July 2012 - Vol. 2; No 2

Abstract

Expected monetary value (EMV) or risked net present value (NPV) is the basis used in this study to maximize well utilization with a focus on evaluating optimal sidetrack time. Sidetrack from a production well (and injection well) depending on the recovery mechanism is meant to exploit by-passed reserves and/or isolated marginal reserves. Production from the main wellbore exploiting a lower horizon is commingled with that from a secondary wellbore (sidetrack) exploiting a higher horizon. The problem can be adapted to sequential production from both horizons. Decision analysis, Monte-Carlo Simulation, time value of money and production forecast by Arps' rate-time model and 1-Dimensional frontal displacement model are applied in-view of optimizing sidetrack time in a spreadsheet. Uncertainty of the effect of the sidetrack operation on production and that of economic and reservoir parameters were incorporated into the analysis. Optimal sidetrack time is highly dependent on the probability of success of the sidetrack while the decision to sidetrack is more profitable over the no-sidetrack option for the cases studied. For application to multi-well systems, a tank-model approach may be favorable, consisting of multi-tank model based on a transfer coefficient for inter-tank interaction/transmissibility.

1. Introduction

The economic viability of a project may solely determine the onward execution of the project without prejudice to political sentiments, but without a negative impact on the environment. Likewise, the risks involved are enumerated and evaluated for mitigation. *EIA*[1] classified risk in the energy sector into 5 groups namely market or price risk, operational risk, credit/default risk, liquidity risk and political risk (new regulation, expropriation). The first three risks, in addition to the basic reservoir parameters affecting production performance shall be considered.

Maximizing recovery of existing producing field is pertinent to improving return-on-investment. Hence, maximizing recovery by either recompleting a well to add more drainage points (extension of perforation zones) or sidetracking through an existing well to establish a secondary wellbore for sequential or commingled production shall be considered. In Nigeria, the Directorate of Petroleum Resources (DPR) regulation does not support commingling of fluids having different fluid properties (PVT properties), so for such cases sequential production is required. Though, smart (intelligent) wells enable both sequential and commingled production with minimal intervention for improved net present value (NPV). DPR commingling exemptions based on permission through the deployment of a smart well[2]. Likewise, marginal field operators in Nigeria are exempted by DPR from the prohibition of commingling for improved NPV due to

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[1] Energy Information Administration, Derivatives and Risk Management in the Petroleum, Natural Gas, and Electricity Industries, U.S. Department of Energy, Washington, DC (2002)

[2] Brock, W. R., Oleh, E. O., Linscott, J. P., and Agara, S.: "Application of Intelligent Completion Technology in a Triple-Zone Gravel-Packed Commingled Producer," paper SPE 101021 presented at the 2006 SPE Annual Technical Conference and Exhibition, San Antonio, Texas, 24-27 September.

marginal accumulation as an incentive[3]. *Konopczynski et al.*[4] revised international regulatory bodies' reason for prohibiting commingling:

- Potential to sub-optimal recovery
- Accurate flow estimation and allocation for each zone
- Fluid compatibility
- Minimization of intervention
- Lack of control (cross flow between reservoirs)

The drainage points along the primary or secondary wellbore is targeted at unlocking the potentials of the field. *Ross et al.*[5] posited that 40% of oil initially in place will be produced from available technology from existing reservoirs and 20% shall be by-passed, 20% in remaining oil columns and 20% as residual oil saturation. Of the by-passed oil and remaining oil in columns, 50% of these shall be economically produced with low cost drilling schemes and slim-hole completion. Reasons for remaining oil lies in reservoir heterogeneity, reservoir compartmentalization and evidence from 4D seismic coverage of the reservoir that was monitored over time to reveal un-drained zones. Additional drainage points are meant to tap remaining oil, by-passed oil or untapped horizons due to less recoverable oil, low permeability or high sulphur content.

Orodu and Tang[6] and *Lerche and Mudford*[7] had evaluated optimal sidetrack (recompletion) time under risk to exploit a less productive horizon with the continued or botched production scenario of the highly productive horizon under production. The former authors studied secondary recovery mechanism under water flooding and the latter primary recovery. Optimal time was evaluated by maximizing an objective function with respect to expected monetary value (EMV) that incorporated the technical and geological risks involved with the sidetrack (recompletion) operation and the uncertainties associated with fiscal regime and production forecast. The work by *Lerche and Mudford*[8] is re-evaluated to consider the impact of reservoir variables on production forecast[9] and optimal sidetrack time, as well as typical and common parameters for evaluating NPV, and estimates of economic parameters suitable to the Niger Delta region. This

[3] Avuru, A.: Comments at the 2011 SPE Annual Oloibiri Lecture and Energy Forum Series, 30 June.
[4] Konopczynski, M., Ajayi A., and Leigh-Ann R.: "Intelligent Well Completion Status and Opportunities for developing Marginal Reserves," paper SPE 85676 presented at the 2003 SPE Nigerian Annual International Conference and Exhibition, Abuja, Nigeria, 4-6 August.
[5] Ross, B. R., Faure, A. M., Kitsios, E. E., Oosterling, P., and Zettle, R. S. "Innovative Slim-hole Completions," paper SPE 2481 presented at the 1992 European Petroleum Conference, Cannes, 16-18 November.
[6] Orodu, O. D. and Tang. Z.: "Risk Evaluation Production-Injection Recompletion and Sidetrack," *Energy Exploration and Exploitation* (2011) 29, No. 3., 235-250.
[7] Lerche, I. and Mudford, B.: "Risk Evaluation for Recompletion and Sidetrack Developments in Oil Fields," paper SPE 71418 presented at the 2001 SPE Annual Technical Conference and Exhibition, New Orleans, 30 September - 2 October.
[8] Ibid
[9] Fetkovich, M. J.: "Decline Curve Analysis using Type Curves," *JPT* (1980) 32, No. 6, 1065-1077

entails the use of Arps' rate-time exponential model for prediction under primary recovery. For Orodu and Tang[10] and Orodu et al.[11], the objective function is modified.

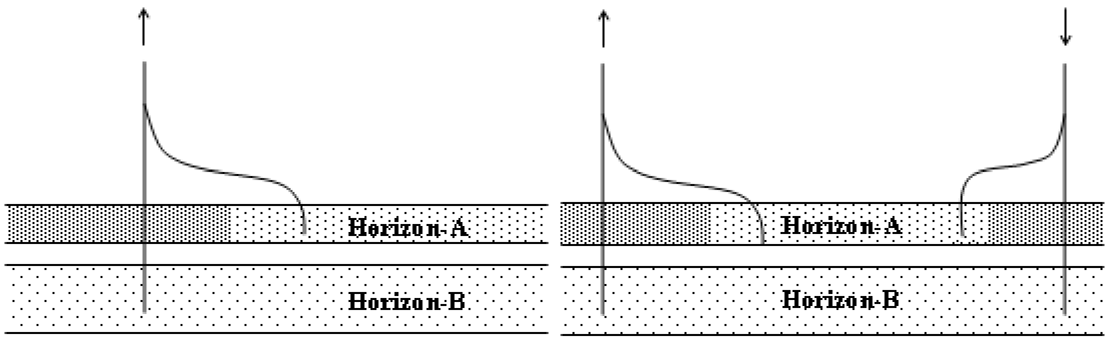

Figure 1: (a) subsurface schematic with production well under natural depletion (b) subsurface schematic with production and injection wells under secondary recovery by water-flooding.

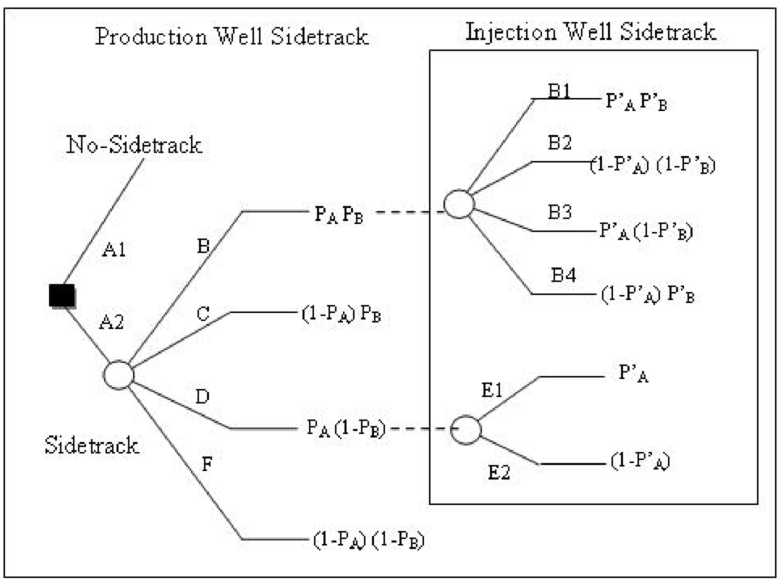

Figure 2: Production well and production-injection well sidetrack decision tree diagram[12]

The subsurface schematic of Figure 1 is adopted for a production well under natural depletion (primary recovery) as shown in Figure 1a and Figure 1b for secondary recovery by water flooding. Based on recovery mechanism and optimization of sidetrack (recompletion) time by EMV, the decision tree of Figure 2 summarizes the associated possible outcomes as regards to the technical and geological uncertainties summed up in probability of success (P_A, P_B, P'_A and P'_B) due to the sidetrack operation. P_A and P_B are the probabilities of success (POS) of production

[10] Orodu and Tang, op. cit.
[11] Orodu, O. D., Ako, C. T., and Craig, A. J.: "Risk assessment of Optimal Sidetrack Time," *Petroleum Science and Technology* (In-Press 2011)
[12] Orodu and Tang, op. cit.

from Horizon-A and continuous production from Horizon-B due to the sidetrack (recompletion) operation to Horizon-A for the secondary wellbore. Horizon-B is under production prior to sidetrack or recompletion. P'_A and P'_B stands for POS associated with the injection well for the probability of successful injection into Horizon-A and continual injection to Horizon-B. Horizon-B represents the horizon under injection prior to sidetrack into Horizon-A. Whereas optimal sidetrack time (t_R) from the production well was evaluated for the primary recovery case (Case-I), optimal sidetrack time for both production well (t_R) and injection well (t_{RR}) was considered for the secondary case (Case II). However, for Case II sequential and simultaneous sidetrack (recompletion) of the production and injection wells was evaluated.

2. Model Formulation

Optimal sidetrack (recompletion) time is evaluated based on maximizing EMV that is the objective function. EMV, the product of NPV and probability of possible outcomes is computed for Case-I and Case-II for primary and secondary oil recovery systems respectively. The possible outcomes are presented in Figure 2. Sum of discounted yearly net cash-flow, NPV, is computed with respect to the components of Figure 3.

A stochastic approach for computing the optimal time is applied. This is as a result of the uncertainty of the POS estimates, cost components, discount rate, and physical and measurable reservoir variables depending on the production performance scheme used.

2.1 Primary Recovery – Optimal Sidetrack Time

The objective function which is EMV, the probability weighted value of net cash flow is optimized. The set-up of the decision tree and its analysis in computing EMV are detailed and as seen in *Newendorp and Schuyler*[13]. Equation (1) is the EMV of the production well sidetrack.

$$E_s = P_A P_B (B) + P_A (1 - P_B) (D) + P_B (1 - P_A) (C) + (1 - P_A). (1 – P_B) (F) \qquad (1)$$

where:

$B = V_B + V_A$

$D = V_B(t_R) + V_A$

$C = V_B – C_{AO}$

$F = V_B(t_R) - C_{AO}$

$$E_s = P_A (V_A + C_{AO}) + P_B[V_B – V_B(t_r)] + V_B(t_r) – C_{AO}. \qquad (2)$$

where B, D, C and F are the NPV associated with the branches on figure 2, P_A is POS of sidetrack from the production well, P_B is POS of having continuous production from Horizon-B

[13] Newendorp, P. and Schuyler J.: *Decision Analysis for Petroleum Exploration*, 2nd ed., Planning Press, Colorado (2000).

after sidetracking, V_A is NPV of Horizon-A, \$MM, V_B is NPV of Horizon-B, \$MM, $V_B(t_R)$ is NPV at optimal sidetrack time for the production well, \$MM, C_{AO} is cost of sidetrack (drilling + completion), \$MM.

NPV, an integral component of EMV is derived based on the components of figure 3. The cumulative oil produced between two time intervals is based on Arps' exponential decline model. Volume of oil produced from Horizon-B over time interval is given in equation (3).

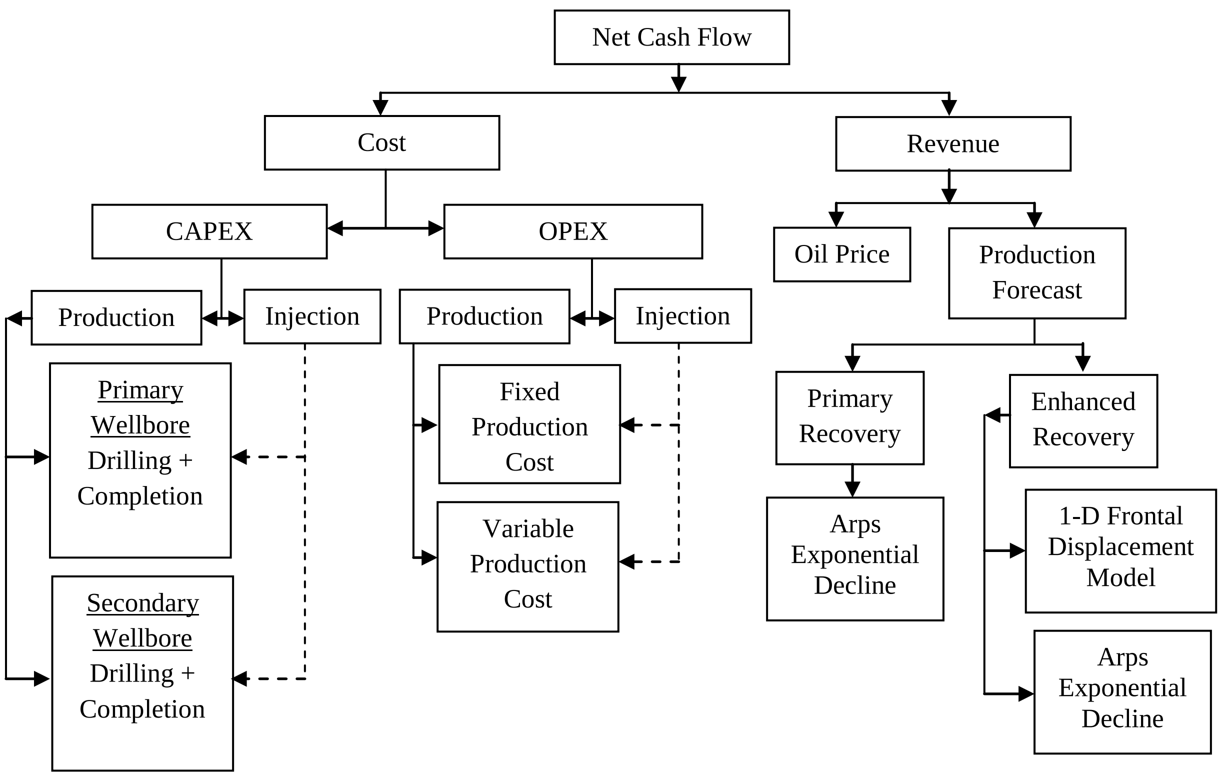

Figure 3: Components of the cash flow

$$dq_B = q_{BO}e^{(-D_B t)}dt \qquad (3)$$

where q_B is volume of oil produced from Horizon-B, bbl; q_{BO} is initial oil production rate from Horizon-B, bbl/yr; D_B is annual nominal exponential decline of Horizon-B and t is time in years.

Given that P_O is oil price, α_B is the variable cost for producing hydrocarbons from Horizon-B (\$/bbl), β_B is fixed production cost as a percent of CAPEX and C_{BO} is the investment required for drilling and completions therefore for a given time interval, NPV is given as,

$$dV_B = (P_o - \alpha_B)q_{BO}e^{-D_B t}e^{-i_D t}dt - C_{BO} - C_{BO}\beta_B e^{-i_D t}dt \qquad (4)$$

$$V_B = (P_o - \alpha_B) q_{BO} e^{-i_D t_R} \int_0^{t_{Bel}} e^{-(D_B+i_D)t}\, dt - C_{BO} - C_{BO}\beta_B \int_0^{t_{Bel}} e^{-i_D t}\, dt \qquad (5)$$

$$V_B = \frac{(P_o-\alpha_B)}{-(D_B+i_D)} q_{BO}\left[e^{-(D_B+i_D)t_{Bel}} - 1\right] - C_{BO} + \frac{C_{BO}\beta_B e^{-i_D t_{Bel}}}{i_D} - C_{BO}\beta_B \qquad (6)$$

where i_D is discount rate and t_{Bel} is time to economic production limit for Horizon-B. Similarly, we can represent the value obtained from Horizon-A at the economic limit (t_{Ael}) as,

$$V_A = \frac{(P_O-\alpha_A)q_{AO}e^{-i_D t_R}}{-(D_A+i_D)}\left[e^{-(D_A+i_D)t_{Ael}} - 1\right] - C_{AO}e^{-i_D t_R} + \frac{C_{AO}\beta_A e^{-i_D t_{Ael}}}{i_D} - \frac{C_{AO}\beta_A e^{-i_D t_R}}{i_D} \qquad (7)$$

Continuous discounting is applied and in particular, the difference between equation (5) and (7) is the production start time from Horizon-A and Horizon-B and so influences the discounting of net cash flow to the present value. Substituting equations (5) and (7) into equation (1) gives equation (8).

$$E_s = \left\{\frac{P_A(P_O-\alpha_A)q_{AO}e^{-i_D t_R}e^{-(D_A+i_D)t_{Ael}}}{-(D_A+i_D)} + \frac{P_A(P_O-\alpha_A)q_{AO}e^{-i_D t_R}}{(D_A+i_D)} + \frac{P_A C_{AO}\beta_A e^{-i_D t_{Ael}}}{i_D} - \frac{P_A C_{AO}\beta_A e^{-i_D t_R}}{i_D}\right\} +$$

$$\left\{\left(\frac{P_B(P_O-\alpha_B)q_{BO}e^{-(D_B+i_D)t_{Bel}}}{-(D_B+i_D)} + \frac{P_B(P_O-\alpha_B)q_{BO}}{(D_B+i_D)} - P_B C_{BO} + \frac{P_B C_{BO}\beta_B e^{-i_D t_{Bel}}}{i_D} - P_B C_{BO}\beta_B\right) +\right.$$

PBPO−αBqBOe−DB+iDtRDB+iD−PBPO−αBqBODB+iD+PBCBO−PBCBOβBe−iDtRiD+PBCB

OβB+PO−αBqBOe−DB+iDtR−DB+iD+PO−αBqBODB+iD−CBO+CBOβBe−iDtRiD−CBOβB−

$$C_{AO}e^{-i_D t_R} \qquad (8)$$

Optimal production well sidetrack time (t_R) is obtained by differentiating equation (8) and making t_R the subject of formula (equation (9)).

$$t_R = -\frac{1}{D_B}\ln\frac{-K}{M} \qquad (9)$$

where,

$$K = \left[\frac{i_D P_A(P_O-\alpha_A)q_{AO}e^{-(D_A+i_D)t_{Ael}}}{(D_A+i_D)} - \frac{i_D P_A(P_O-\alpha_A)q_{AO}}{(D_A+i_D)} + i_D C_{AO} + P_A C_{AO}\beta_A + P_B C_{BO}\beta_B - C_{BO}\beta_B\right] \qquad (10)$$

$$M = q_{BO}(P_O - \alpha_B)(1 - P_B) \qquad (11)$$

In addition, Fetkovich's[14] derivation of Arps' exponential decline, equations (12) and (13) are applied to study the impact of reservoir variables on EMV as applied by Orodu and Tang[15].

$$\frac{q(t)}{q_i} = e^{-[(q_i)_{max}/Np_i]t} \tag{12}$$

$$\frac{(q_i)_{max}}{Np_i} = \frac{0.00634k}{\varphi\mu C_t r_w^2}\frac{1}{\frac{1}{2}\left[\left(\frac{r_e}{r_w}\right)^2-1\right]\left[\ln\left(\frac{r_e}{r_w}\right)-\frac{1}{2}\right]} \tag{13}$$

where q(t) is production rate at time t; q_i is initial production rate; Np_i is cumulative production at time t; φ is porosity; μ is viscosity; C_t is compressibility; r_w is wellbore radius and r_e is reservoir boundary radius. On comparing equation (12) with the Arps' exponential decline model, decline rate (D_A and D_B) in the optimal sidetrack equation can be represented by equation (13). The NPV for the no-sidetrack option is obtained from equation (5). The option to sidetrack shall be compared with the option not to sidetrack. The spreadsheet of a commercial software for decision, risk and economic analysis is used.

2.2 Secondary Recovery – Optimal Sidetrack Time

While optimal sidetrack time for the primary recovery case is based on the production well sidetrack portion of figure 2, that of the secondary recovery case under waterflood is based on the combination of production and injection well sidetrack. Two production performance schemes are applied, Arps' exponential decline model as applied in the previous case for production under natural depletion and Yang's model[16] based on 1-D Buckley-Leverett frontal displacement theory. The Yang's model predicts oil production under waterflood and its efficacy in predicting performance at water-cut greater than 50% and in particular from the water breakthrough point onwards is presented in Orodu et al.[17] when compared to performance prediction by a numerical reservoir simulator. The exponential decline model is applied for those scenarios (outcomes) where continuous water injection failed (Branch-B2 and B3, see figure 2) or injection sidetrack into Horizon-A was a failure (Branches B2, B4 and E2). And during the time lag between production well sidetrack and injection well sidetrack for production from Horizon-A for properly modeling the sequential sidetrack scenario, the exponential decline model is also used. In actual sense, the decision tree depicts sequential production and injection sidetrack. The model to be developed shall consider sequential and simultaneous sidetrack from both wells.

[14] Fetkovich, op. cit.
[15] Orodu and Tang, op. cit.
[16] Yang, Z.: "A New Diagnostic Analysis Method for Waterflood Performance," *SPE Reservoir Evaluation &* Engineering (2009) 12, No. 2, 341-351
[17] Orodu, O.D., Tang, Z., and Anawe, P. A. L.: "Sidetrack and Recompletion Risk Evaluation – Waterflooded Reservoir", *Journal of Petroleum Science and Engineering* (2011) 78, No. 3-4, 705-718

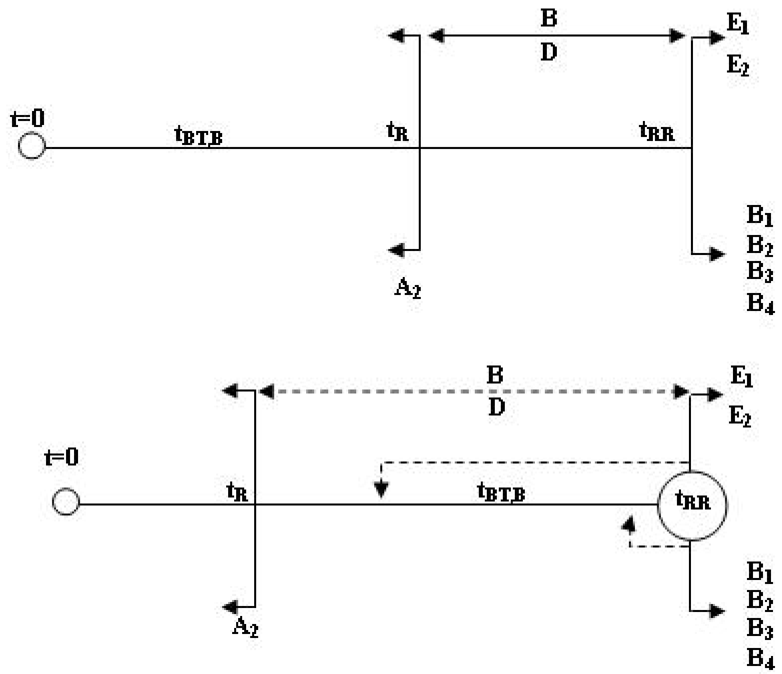

Figure 4: Production timeline for Zone-B indicating production well sidetrack time (t_R) and injection well sidetrack time (t_{RR}) scheme adopted for all branches (a) Post water breakthrough time ($t_{BT,B}$) sidetrack of Zone-B (b) Pre water breakthrough time ($t_{BT,B}$) sidetrack of Zone-B with the possibility of injection well sidetrack occurring before and after water breakthrough time of Zone-B[18].

Figure 4 shows the timeline for production from Horizon-B with sidetrack times indicated on it. The figure further shows the flexibility of production sidetrack occurring before and after water breakthrough time in the production well.

Oil production forecast under water flooding is broken down to equation (14)

$$q_o = \frac{q_L}{2}\left[1 - \sqrt{1 - 4\left(\frac{E_v}{B}\right)\left(\frac{PV}{N_L}\right)}\right] \qquad (14)$$

This is based on Yang's model where,

$$Y = \left(\frac{E_v}{B}\right)\frac{1}{t_D} \qquad (15)$$

$$f_o = {}^{1}/_{2}\left(1 - \sqrt{1 - 4Y}\right) \qquad (16)$$

[18] Orodu and Tang, op. cit.

Equation (14) is further broken down to,

$$q_o = \frac{i_L}{2}\left[1 - \sqrt{1 - 4\left(\frac{E_v}{B}\right)\left(\frac{PV}{ti_w}\right)}\right] \quad (17)$$

where N_L is approximately equal to I_W for time dependency which accounts for constant water injection rate, hence $I_W=t \times i_w$; q_o is oil rate; q_L, liquid rate; PV, pore volume; N_L, cumulative liquid production; E_V, volumetric sweep efficiency; B, relative permeability ratio parameter; t_D, fraction of cumulative liquid production to related formation volume; Y, oil fractional flow equation; fo, oil-cut; I_w, cumulative water injection; i_w, water injection rate.

Equation (18) gives the EMV (objective function for optimization) for either the simultaneous production-injection sidetrack or the sequential production-injection well sidetrack for the waterflood reservoir case. The equation is based on the decision tree of figure 2.

$$Es = P_B P_A\left[(B_1 + B + A_2)\acute{P}_B\acute{P}_A + (B_2 + B + A_2)\left(1 - \acute{P}_B\right)\left(1 - \acute{P}_A\right) + (B_3 + B + A_2)\left(1 - \acute{P}_B\right)\acute{P}_A + (B_4 +\right.$$

$$B{+}A2PB1{-}PA{+}PB1{-}PAC{+}A2{+}1{-}PBPAE1{+}D{+}A2PA{+}E2{+}D{+}A21{-}PA{+}1{-}PB1{-}PAA2$$

(18)

The equation is after *Orodu and Tang* [19] with timelines shown in figure 4. Based on the components of the net cash flow schematic of figure 3 and the assumption of uniform oil production before water breakthrough which was verified by numerical simulation of 1D flow, the NPV for the no-sidetrack scenario is given in equation (19).

$$NPV_{ns} = -C_B - \acute{C}_B - I_B i_w \int_0^{t_{B98}} e^{-i_d t}\, dt + (P_o - \alpha_B)\int_{t_{BT,B}}^{t_{B98}} q_{1B} e^{-i_D t}\, dt + \frac{(P_o - \alpha_B)}{t_{BT,B}} N_{P\,BT}\int_0^{t_{BT,B}} e^{-i_D t}\, dt -$$

$$\beta\left(C_B + \acute{C}_B\right)\int_0^{t_{B98}} e^{-i_D t}\, dt \quad (19)$$

where in equation (18) and (19) A, A_2, B, B_2, C, D, E_1 and E_2 are NPV linked to the branches on the decision tree of figure 2 ; P_A and P'_A are the POS of production and injection wells for Horizon-A; P_B and P'_B are the POS of production and injection wells for Horizon-B; i_D is discount rate; Po is oil price; $t_{BT,B}$ is water breakthrough time for Horizon-B; t_{B98} is time to 98% water-cut production for Horizon-B; β is fixed production (injection) cost; I_B is variable injection cost per barrel; i_w is water injection rate; α_B is variable production cost for Horizon-B; $Np_{BT,B}$ is cumulative oil produced at $t_{BT,B}$; q_{1B} is oil production rate based on waterflood performance; C_B is production well cost; C'_B is injection well cost; q_{1B} is production rate subject to Yang's model for Horizon-B.

[19] Ibid

The optimization function for evaluating the optimal sidetrack time is equation (18). For clarity, the NPV equations for Branches A2, B, B1 which represents the decision tree path ~~A2~~→ B→B1 are stated below, which are similar to equation (19) but presented to cater for the timelines of figure 4a ($t_{BT,B}<t_R<t_{max}$) and figure 4b ($0<t_R<t_{max}$). Solution to the objective function is by using Solver in a spreadsheet. The constraints include watercut of production from Horizon-B being less than 98% ($f_w \leq 98\%$) and injection well sidetrack time is greater than or equal to production well sidetrack time ($t_{RR} \geq t_R$).

$$
\begin{aligned}
A_2(t_{BT,B} < t_R < t_{max}) \\
&= -C_B - C_A e^{-i_D t_R} - \grave{C}_B \\
&- \beta(C_B + \grave{C}_B) \int_0^{t_R} e^{-i_D t} dt - I_B i_w \int_0^{t_R} e^{-i_D t}\, dt + (P_o - \alpha_B) \int_{t_{BT,B}}^{t_R} q_{1B} e^{-i_D t} dt \\
&+ \frac{N_{P\,BT,B}(P_o - \alpha_B)}{t_{BT,B}} \int_0^{t_{BT,B}} e^{-i_D t} dt
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
A_2(t_{BT,B} < t_R < t_{max}) \\
&= -C_B - C_A e^{-i_D t_R} - \grave{C}_B \\
&- \beta(C_B + \grave{C}_B) \int_0^{t_R} e^{-i_D t} dt - I_B i_w \int_0^{t_R} e^{-i_D t}\, dt + \frac{N_{P\,BT,B}(P_o - \alpha_B)}{t_{BT,B}} \int_0^{t_R} e^{-i_D t} dt
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
B(t_{BT,B} < t_R < t_{max}) \\
&= -\beta \left[C_B \int_{t_R}^{t_{RR}} e^{-i_D t} dt + C_A^R \int_{t_R}^{t_{RR}} e^{-i_D t} dt \right] \\
&- I_B i_w \int_{t_R}^{t_{RR}} e^{-i_D t}\, dt + (P_o - \alpha_{1A}) e^{-i_D t_R} \int_0^{t_{RR} - t_R} q_{2A} e^{-(D_A + i_D)t}\, dt \\
&+ (P_o - \alpha_B) \int_{t_R}^{t_{RR}} q_{1B} e^{-i_D t}\, dt
\end{aligned}
\tag{22}
$$

$B(0 < t_R < t_{max})$

$$= -\beta\left[C_B \int_{t_R}^{t_{RR}} e^{-i_D t} dt + C_A^R \int_{t_R}^{t_{RR}} e^{-i_D t} dt\right]$$

$$- I_B i_w \int_{t_R}^{t_{RR}} e^{-i_D t}\, dt + (P_o - \alpha_{1A}) e^{-i_D t_R} \int_0^{t_{RR}-t_R} q_{2A} e^{-(D_A+i_D)t}\, dt$$

$$+ (P_o - \alpha_B) \int_{t_{BT,B}}^{t_{RR}} q_{1B} e^{-i_D t}\, dt + \frac{N_{P\,BT,B}(P_o - \alpha_B)}{t_{BT,B}} \int_{t_R}^{t_{BT,B}} e^{-i_D t} dt \tag{23}$$

$B_1(t_{BT,B} < t_R < t_{max}; 0 < t_R < t_{max})$

$$= -\beta\left[\left(C_B + \grave{C}_B\right) \int_{t_{RR}}^{t_{B98}} e^{-i_D t} dt + \left(C_A^R + \grave{C}_A\right) \int_{t_{RR}}^{t_{A98}} e^{-i_D t} dt\right] - \grave{C}_A e^{-i_D t_{RR}} - I_A i_w \int_{t_{RR}}^{t_{A98}} e^{-i_D t}\, dt$$

$$- I_B i_w \int_{t_{RR}}^{t_{B98}} e^{-i_D t}\, dt + (P_o - \alpha_{1A}) e^{-i_D (t_{RR})} \int_{t_{BT,A}}^{t_{A98}} q_{1A} e^{-i_D t}\, dt + (P_o - \alpha_B) \int_{t_{RR}}^{t_{B98}} q_{1B} e^{-i_D t}\, dt$$

$$+ \frac{N_{P\,BT,A}(P_o - \alpha_{1A})}{t_{BT,A}} e^{-i_D t_{RR}} \int_0^{t_{BT,A}} e^{-i_D t} dt \tag{24}$$

where t_R is production well sidetrack time; t_{RR} is injection well sidetrack time (t_{RR}>t_R); I_A is variable injection cost per barrel for Horizon-A; C_A is production well sidetrack cost for Horizon-A; C'_A is injection well sidetrack cost for Horizon-A; q_{2B} is initial oil production rate subject to Arps' exponential decline model; i_{wA} is constant water injection rate to Horizon-A; α_A is variable production cost, Horizon-A; $t_{BT,A}$ is water breakthrough time for horizon-A; t_{A98} is water-cut at 98% water production from Horizon-A; $Np_{BT,A}$ is cumulative oil production at $t_{BT,A}$.

3. Results

The parameters and its associated deterministic and stochastic values are presented in Tables A1, A2 and A3 in the Appendix for production under primary recovery and that of sequential sidetrack from both production and injection wells for secondary recovery under waterflood displacement. For the former, production from Horizon-B depicts a typical high profile production rate in the Niger Delta region. Well sidetrack cost (drilling + completion) also is typical of onshore Niger Delta estimates. The drilling plus completion cost in Table A2 for the sequential production-injection well sidetrack for secondary recovery are typical estimates from Asia (China). Injection well cost estimate depicted in Table 2 is a quarter of regular cost estimate.

This signifies that the injection well is meant to improve performance in an inverted 5-spot pattern. The reservoir and well parameters for the secondary recovery are obtained from performance of a synthetic field.[20].

3.1 Primary Recovery

Two cases are considered, Case-1 involves stochastic parameters of economic and POS, and Case-2 involves economic, POS and reservoir-well parameters. The NPV statistics of the no-sidetrack option for mean, minimum, maximum and standard deviation are \$1228.2MM, \$767.1MM, \$1674.4MM and \$1.8MM respectively. This indeed is profitable but the analysis entails a look ahead scenario computation without exploration cost[21]. EMV of sidetrack option has a statistics comprising of mean, minimum, maximum and standard deviation of \$1493.8MM, \$914.9MM, \$2188.1MM and \$2.2MM respectively. The overlap graph of NPV and EMV distribution is shown in figure 5. Figure 5 clearly shows the profitability of the sidetrack option over the no-sidetrack option. Optimal sidetrack time (t_R) distribution is characterized by minimum, maximum, mean, standard deviation and standard error of 1.67yrs, 4.04yrs, 2.88yrs, 0.36yrs and 0.00 respectively (figure 6). The rank correlation of the parameters with respect to t_R gives the relative influence of each parameter, and it is in the decreasing order of P_B, P_A, i_D, C_{AO}, α_B, α_A…… (figure 7a). The other parameters show minimal or insignificant influence on t_R.

Case-2 incorporates the stochastic tendency of reservoir and well parameters to the parameters used for Case-1. The rank correlation of parameters to t_R is different (figure 7b). in decreasing order of relevance, the parameters are r_e_Horizon-B, μ_Horizon-B, P_B, k_Horizon-B, Ct_Horizon-B, r_e_Horizon-A, μ_Horizon-A, P_A …… Oil price was the least significant. This shows that reservoir parameters of Horizon-B influences productivity and in turn the volume of producible reserves. Hence, the reservoir parameters and POS are the most vital parameters. All the parameters are estimates to a certain degree of accuracy.

[20] Orodu, Tang and Anawe, op. cit.
[21] Kue Y. N. and Orodu O. D.: “Economic Analysis of Innovative Approaches to Marginal Field Development,” paper SPE 106001 presented at the 2006 SPE Nigeria Annual International Technical Conference and Exhibition in Abuja, Nigeria, 31 July- 2 August.

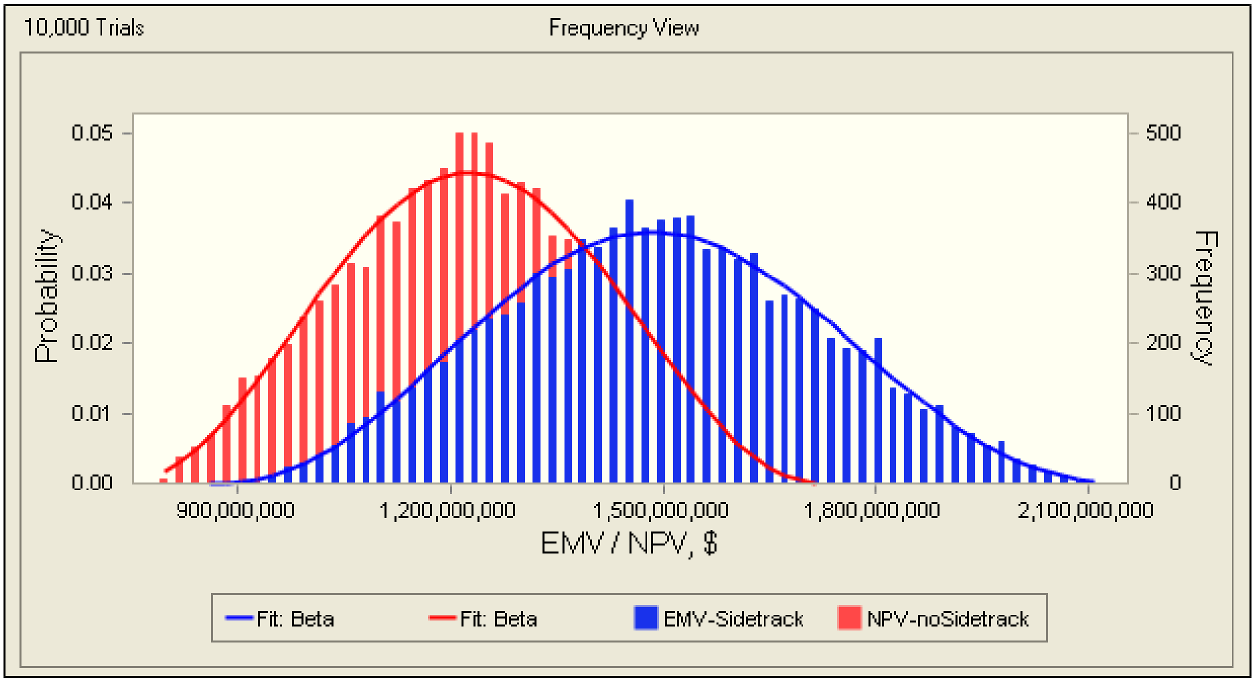

Figure 5: Distribution and statistical fitness curve of NPV and EMV for the no-sidetrack and sidetrack options.

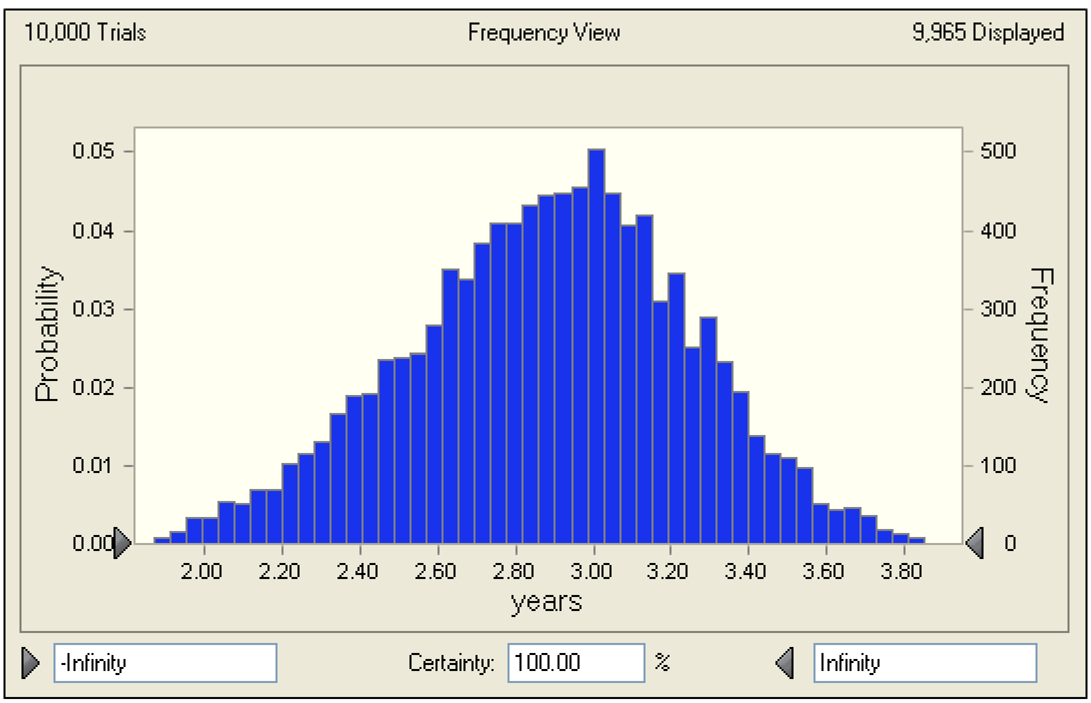

Figure 6: Optimal production well sidetrack time (t_R) distribution

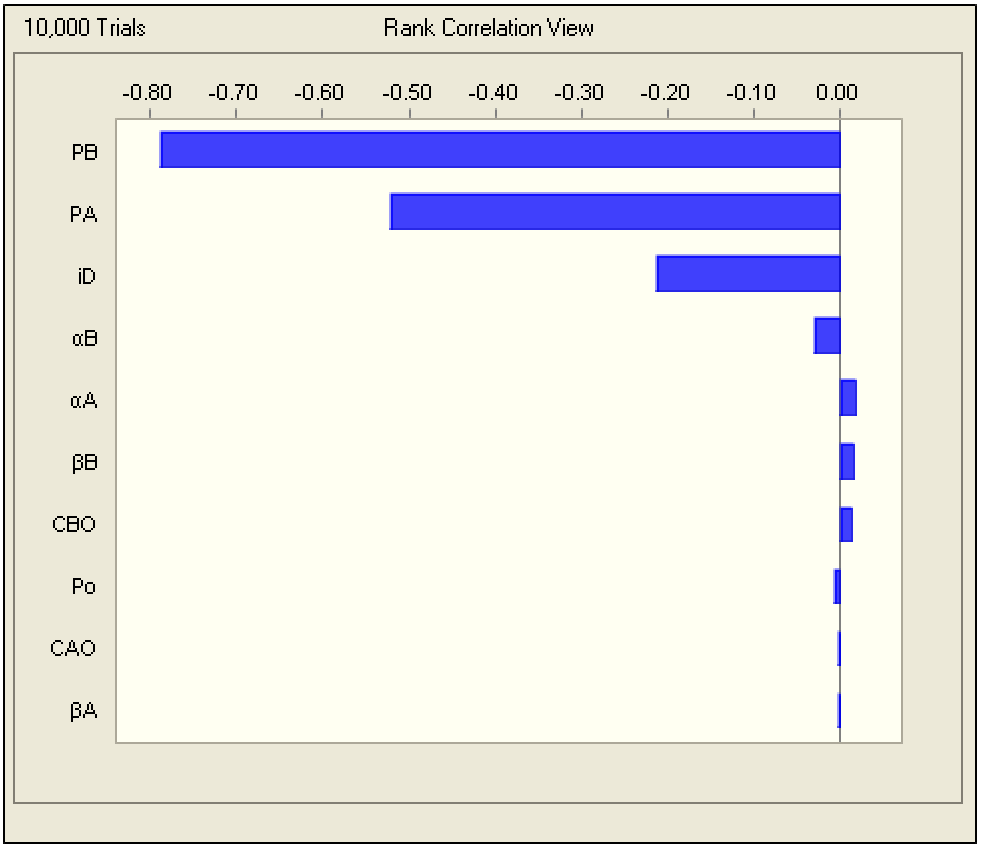

Figure 7a: Rank correlation of stochastic parameters with optimal time (t_R) Case-1

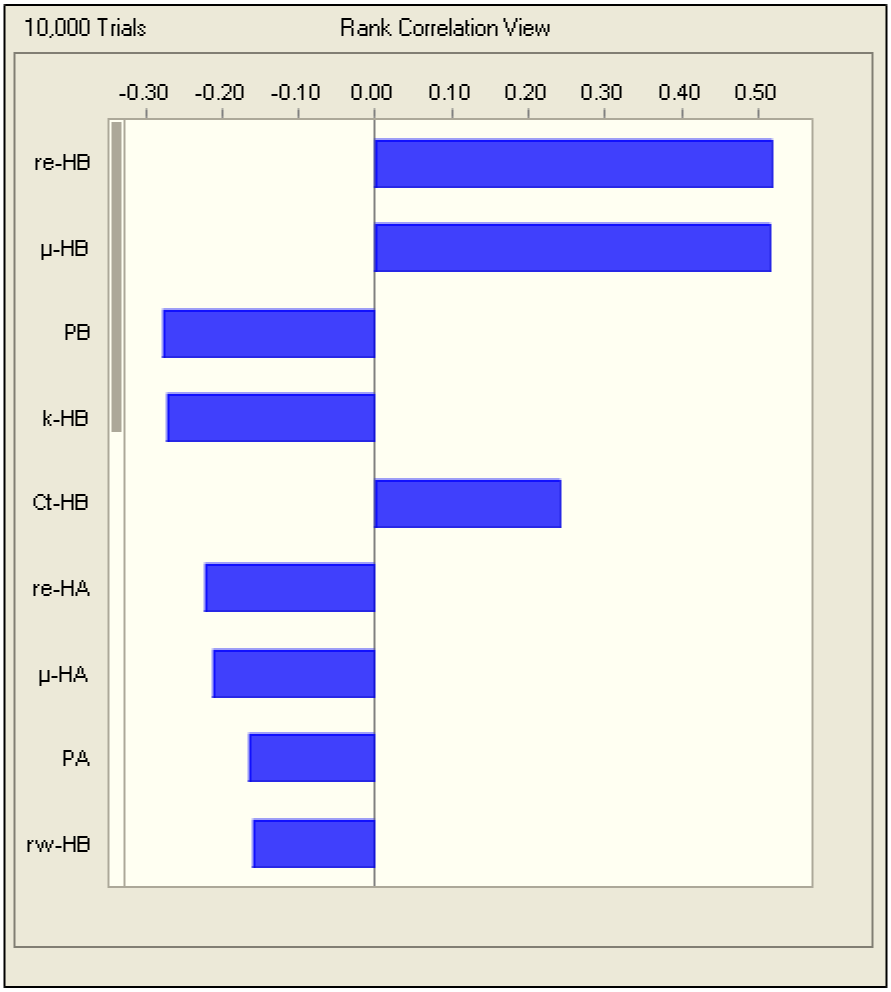

Figure 7b: Rank correlation of stochastic parameters with optima; time (t_R) Case-2

3.2 Secondary Recovery – Production and Injection Wells Optimal Sidetrack Time

Summary statistics of minimum, maximum, mean, standard error and standard deviation for optimal production sidetrack time are 63.92months, 113.36months, 98.76months, 0.19 and 6.16momths respectively (figure 8). Though sequential production and injection sidetrack time was modeled, the results obtained showed the same optimal time for both, thereby ensuring simultaneous production and injection wells sidetrack is preferable when possible. The corresponding EMV and NPV for the sidetrack and no-sidetrack options are shown in an overlap chart (figure 8) with no recognizable superior option. However, figure 10, the cumulative probability plot shows the marginal profitability of the sidetrack option over the no-sidetrack option. Furthermore, the rank correlation of the stochastic parameters to t_R gives a decreasing order of influence according to P'_A, P_A, P'_B, Sw_{BT}_Horizon-A, Ev_Horizon-A, P_B, Swi_Horizon-A,……… (see figure 11). The POS of the sidetrack operation for both wells is vital to the economics of the recovery process. An advantage of the water displacement method used for production prediction is the determination of watercut at optimal time for production from Horizon-B. This is displayed in figure 12 for the 1000 Monte-Carlo Simulation runs with a log-normal distribution and mean of 0.9067. It should be noted that the associated water disposal and treatment cost is not considered, but assumed been part of the fixed and variable production cost.

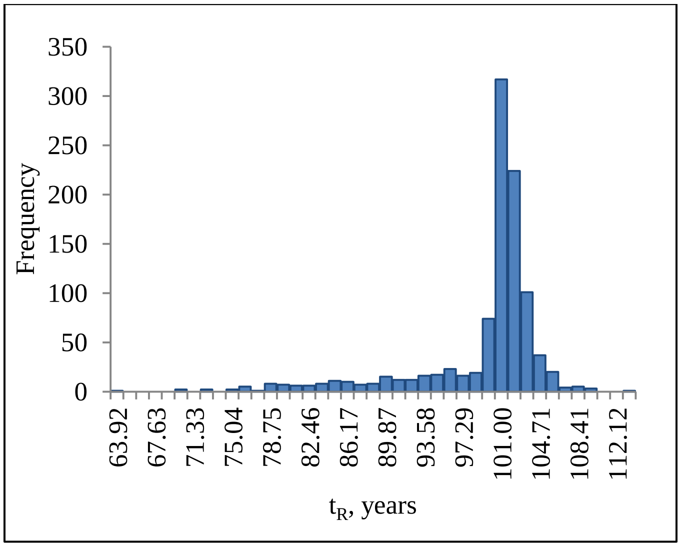

Figure 8: Optimal production-injection well sidetrack time distribution (t_R)

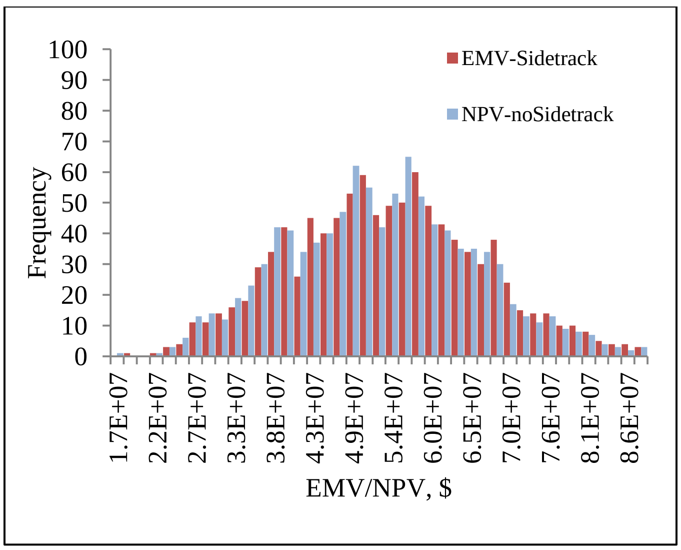

Figure 9: Distribution of EMV and NPV for the sidetrack and no-sidetrack options

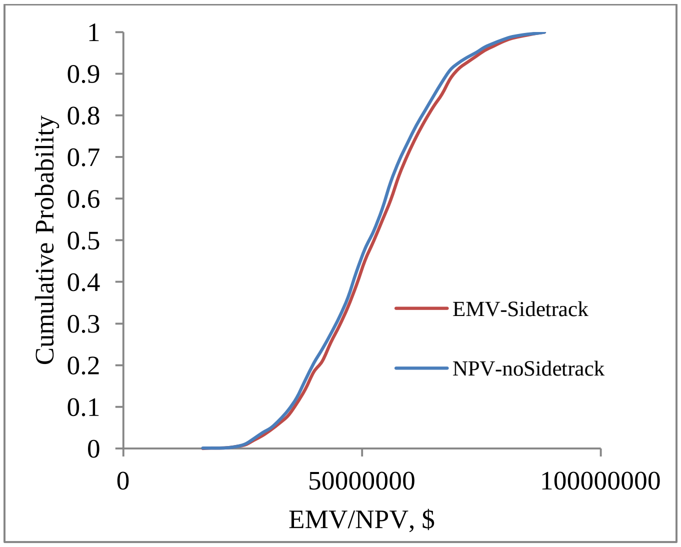

Figure 10: cumulative probability of EMV and NPV for the sidetrack and no-sidetrack options

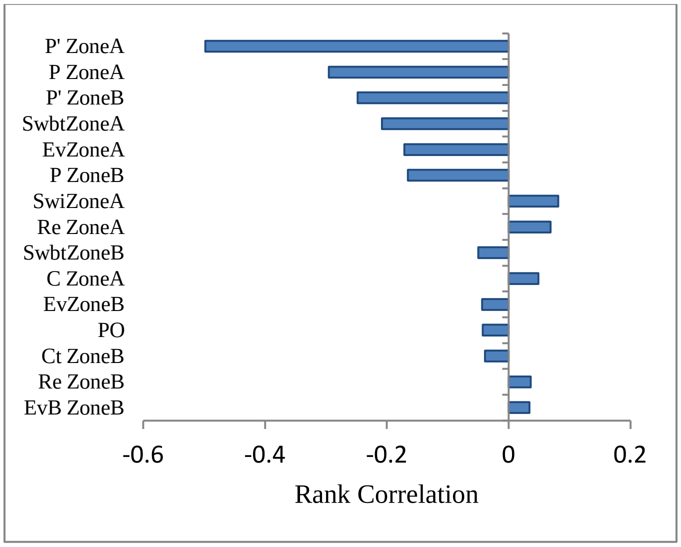

Figure 11: Rank correlation coefficient of stochastic parameters with optimal production-injection well sidetrack time.

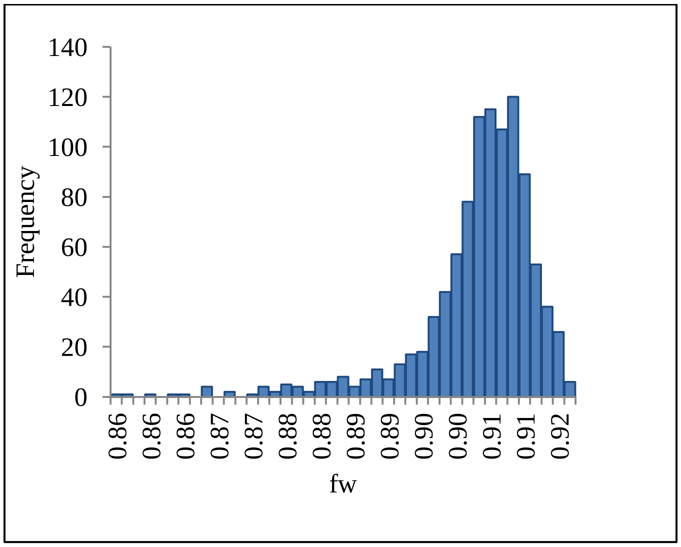

Figure 12: Watercut of production from Horizon-B at t_R.

4. Discussion

Exploration cost is not included as the analysis was used for comparative analysis of the decision to sidetrack or not. So the results obtained are valid but the use for return-on-investment as a tool for decision making should include the finding-cost. For the secondary recovery case, the cost of produced water disposal and treatment was not considered. The mean water-cut at optimal sidetrack time was as high as 0.92 for production from Horizon-B. Therefore, it is necessary to include water treatment and disposal cost.

The Arps' exponential decline model was the basis for predicting oil production for recovery under natural depletion. The model has been tested and verified from theory[22] and other form of Arps' models such as hyperbolic has been applied to water flooding, the model by Yang[23] that was based on Buckley-Leverett's 1-D frontal displacement derivation was verified.[24] . So, the analytical prediction models are suitable for single well systems. For multi-well system, Orodu et al.[25] had recommended material balance application to allocation of injected water for secondary recovery by water flooding[26]. However, for production forecast under any recovery mechanism, the material balance (tank model) approach of multi-well system using multi-tanks is an option. Bitsindou[27] developed a dynamic model that incorporates nodal analysis, material balance and

[22] Fetkovich, op. cit.
[23] Yang, op. cit.
[24] Orodu, Tang and Anawe, op. cit.
[25] Ibid
[26] Chapman. L. R. and Thompson, R.R.: "Waterflood Surveillance in the Kuparuk River Unit with Computerized Pattern Analysis," *JPT* (1989) 41, No. 3, 277-282.
[27] Bitsindou, A. B.: "Matching of Production Performance Using Dynamic Nodal Analysis," PhD dissertation, University of Tulsa, Oklahoma (2002)

decline analysis. The point of interest is that of the multi-tank model having radial regions around the well (well region) and a larger region (reservoir region) with multi-phase flow across the tanks and regions through transfer coefficients. Other means of injected fluid allocation are qualitative. The solution for multiple zones (horizons) is possible, however, complex based on the decision tree schematic and optimization.

Apart from EMV, as a criterion for computing optimal sidetrack time, other methods are subjective in nature due to no particular value but a range of values. These methods are volatility and probability of profit of the sidetrack option over the no-sidetrack option[28].

5. Conclusion

In-view of maximizing well utilization across horizons, by-passed reserves and marginal accumulation, optimal production (and injection) well sidetrack time was evaluated for production under natural decline (primary recovery) and waterflood (secondary recovery), the conclusion drawn are;

1. The probability of success of the sidetrack operation is more vital than other parameters, including oil price.
2. For the case of production and injection well sidetrack for the secondary recovery, simultaneous production-injection well sidetrack is optimal compared to sequential sidetrack.
3. Material balance (tank model) may be the best option to study different production mechanism. In addition, multi-tank model based on transfer coefficient between the tanks is an option for dynamic production forecast of multi-well system due to well interaction.
4. The option to sidetrack was more profitable than the no-sidetrack based on probability density. However, that of secondary recovery required the use of cumulative probability curve due to the indistinguishable overlap of the probability density of both options.

[28] Lerche, I., Noeth, S., 2004, “sidetrack and recompletion risks in oil fields-I Recompletion Decisions based on Estimated Net present Valuue – In Economics of Petroleum Production, A Compendium Volume 1: Profit and Risk, Multi-Science Publishing Co. Ltd, UK, pp 165-191

Appendix – A

Table A1: Economic and Reservoir Parameters (Primary Recovery)

Economic Parameters	Symbol	Minimum	Most Likely	Maximum
Oil Price, $/bbl	P_o	50	75	100
Discount rate, %/year	i_D	0.10	12	0.14
Variable Prod. Cost, Horizon-A, $/bbl	α_A	2.5	3.0	4.5
Variable Prod. Cost, Horizon-B, $/bbl	α_B	2.5	3.0	4.5
Fixed Prod. Cost, Horizon-A, %/year	β_A	0.045	0.05	0.06
Fixed Prod. Cost, Horizon-B, %/year	β_B	0.045	0.05	0.06
Sidetrack Cost, Horizon-A, $MM	C_A	12.0	15.0	17.0
Well Cost, Horizon-B, $MM	C_B	17.0	19.5	21.0
Probability of Success (POS)				
POS Sidetrack, Horizon-A	P_A	0.4	0.6	0.8
POS Sidetrack, Horizon-B	P_B	0.4	0.6	0.8
Production Forecast Variables				
Nominal Exp. Decline, Horizon-A, yr^{-1}	D_A		0.5763	
Nominal Exp. Decline, Horizon-B, yr^{-1}	D_B		0.7186	
Initial Prod. Rate, Horizon-A, bbl/yr	q_A		7,300,000	
Initial Prod. Rate, Horizon-B, bbl/yr	q_B		14,600,000	
Economic Prod. Limit, Horizon-A, yr	t_{Ael}		15.4	
Economic Prod. Limit, Horizon-B, yr	t_{Bel}		13.3	
Reservoir & Well Parameters Horizon-A				
Porosity	Φ	0.15	0.16	0.2
Compressibility, psi^{-1}	C_t		13×10^{-6}	
Viscosity, cP	μ	3	5	9
Well Radius, ft	r_w	0.1	0.584	1
Reservoir Boundary, ft	r_e	490	600	820
Permeability	K	2.5	3.0	4.0
Reservoir & Well Parameters Horizon-B				
Porosity	Φ	0.175	0.180	0.220
Compressibility, psi^{-1}	C_t		13×10^{-6}	
Viscosity, cP	μ	3	5	9
Well Radius, ft	r_w	0.2	0.584	1
Reservoir Boundary, ft	r_e	490	600	820
Permeability	K	4	5	7

Table A2: Economic (Secondary Recovery)

Economic Parameters	Symbol	Minimum	Most Likely	Maximum
Oil Price, $/bbl	P_o	50	75	100
Discount rate, %/month	i_D	0.10	1.005	0.14
Variable Prod. Cost, Horizon-A, $/bbl	α_A		5	
Variable Prod. Cost, Horizon-B, $/bbl	α_B		5	
Variable Inj. Cost, Horizon-A, $/bbl	I_A		2	
Variable Inj. Cost, Horizon-B, $/bbl	I_B		2	
Fixed Prod. & Inj. Cost, Horizon-A, %/month	β_A		0.0025	
Fixed Prod. & Inj. Cost, Horizon-B, %/month	β_B		0.0025	
Sidetrack Cost Prod. Well , Horizon-A, $MM	C_A	1.500	1.900	2.500
Well Cost Prod. Well, Horizon-B, $MM	C_B	2.000	2.800	4.000
Sidetrack Cost Inj. Well , Horizon-A, $MM	C'_A	0.400	0.475	0.900
Well Cost Inj. Well, Horizon-B, $MM	C'_B	0.600	0.700	1.500
Probability of Success (POS)				
POS Sidetrack Prod. Well, Horizon-A	P_A	0.3	0.6	0.8
POS Sidetrack Prod. Well, Horizon-B	P_B	0.3	0.6	0.8
POS Sidetrack Inj. Well, Horizon-A	P_A	0.3	0.6	0.8
POS Sidetrack Inj. Well, Horizon-A	P_A	0.3	0.6	0.8
Production Forecast Variables				
Initial Prod. Rate, Horizon-A, bbl/yr	q_A		7,300,000	
Initial Prod. Rate, Horizon-B, bbl/yr	q_B		14,600,000	
Economic Prod. Limit, Horizon-A, yr	t_{Ael}		15.4	
Economic Prod. Limit, Horizon-B, yr	t_{Bel}		13.3	

Table A3: Reservoir and well parameters (Secondary Recovery)

Natural Decline Horizon-A				
Porosity	φ	0.09	0.12	0.17
Compressibility, psi^{-1}	C_t	10×10^{-6}	13×10^{-6}	17×10^{-6}
Viscosity, cP	μ	3	5	9
Well Radius, ft	r_w	0.1	0.584	1
Reservoir Boundary, ft	r_e	490	600	820
Permeability	k	10	15	20
Natural Decline Horizon-B				
Porosity	φ	0.19	0.25	0.27
Compressibility, psi^{-1}	C_t	10×10^{-6}	13×10^{-6}	17×10^{-6}
Viscosity, cP	μ	3	5	9
Well Radius, ft	r_w	0.2	0.584	1
Reservoir Boundary, ft	r_e	750	800	900
Permeability	k	30	50	60
Waterflood Horizon-A				
Water Breakthrough time, months			52	
Time (watercut=0.94), months	t_{A94}		202	
Time (watercut=0.98), months	t_{A98}		581	
Cumulative Prod. @ t_{BT} , mmbbl	$Np_{BT,A}$		0.313	
Oil-originally in-place, mmbbl	OOIP		1.271	
Pore Volume, mmbbl	PV		1.956	
Initial water saturation	Swi	0.34	0.35	0.36
Water saturation @ t_{BT}	Sw_{BT}	0.6	0.7	0.75
E_V	Ev	0.5	0.65	0.7
E_V/B	EvB	0.0435	0.044	0.445
Waterflood Horizon-B				
Water Breakthrough time, months			46	
Time (watercut=0.94), months	t_{B94}		147	
Time (watercut=0.98), months	t_{B98}		423	
Cumulative Prod. @ t_{BT} , mmbbl	$Np_{BT,B}$		0.904	
Oil-originally in-place, mmbbl	OOIP		2.852	
Pore Volume, mmbbl	PV		4.075	
Initial water saturation	Swi	0.29	0.30	0.31
Water saturation @ t_{BT}	Sw_{BT}	0.55	0.67	0.70
E_V	Ev	0.55	0.70	0.75
E_V/B	EvB	0.045	0.050	0.055